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Admissibility analysis for linear singular systems with time-varying delays via neutral system approach

Zhou-Yang Liu [a,b], Chong Lin [a,*], Bing Chen [a]

[a] Institute of Complexity Science, Qingdao University, Qingdao 266071, PR China
[b] School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, PR China

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ABSTRACT

This paper studies the admissibility problem for a class of linear singular systems with time-varying delays. In order to highlight the relations between the delay and the state, the singular system is transformed into a neutral form. Then, an appropriate type of Lyapunov–Krasovskii functionals is proposed to develop a delay-derivative-dependent admissibility condition in terms of linear matrix inequalities. The derivation combines the Wirtinger-based inequality and reciprocally convex combination method. The present criterion is also for the stability test of retarded and neutral systems with time-varying delays. Some examples are provided to illustrate the effectiveness and the benefits of the proposed method.

1. Introduction

Singular systems can better describe physical models than state-space systems and have extensive applications in economics, circuits, large-scale systems and other areas due to the fact that singular system models include not only dynamic equations but also algebraic constraints [1,2]. Depending on the areas of application, singular systems are also referred to as degenerate, descriptor, generalized, differential-algebraic or semi-state systems [2]. In the past decades, there have been many research papers published to address the importance and the engineering motivation of singular time-delay systems as time-delay is frequently the main cause of instability and poor performance and such systems arise in a variety of physical systems such as flexible arm control of robots, large-scale electric network control and lossless transmission lines [3–12,14–16].

It is noted that admissibility is the fundamental research topic to a singular time-delay system as it is crucial in analysis and control for such type systems. For this reason, recently, increasing efforts have been devoted to admissibility analysis for singular time-delay systems [4–15]. For example, by decomposing the system into fast and slow subsystems and using some Lyapunov–Krasovskii functionals (LKFs), the robust admissibility problem was investigated in [4], where a sufficient condition in terms of linear matrix inequalities (LMIs) ensuring a singular time-delay system to be regular, impulse free and stable was presented. The corresponding results for the commensurate time-delay case can be found in [5]. However, the above admissibility results for singular time-delay systems in [4,5] are delay-independent, and therefore these results are conservative. In order to establish more efficient delay-dependent admissibility criteria for singular time-delay systems through solving the corresponding LMIs, serval attempts have been done including the descriptor system approach [6], free weighting matrix method combined with the Lyapunov function [7] and graph theory [8], delay-decomposition method [9,10], reciprocally convex combination method [11–13] and parameterized LKF method [14]. Very recently, in our work [15], a neutral system approach was reported applying for singular time-invariant-delay systems and the results are advantageous over the existing ones in the literature. However, the method in [15] is applicable only for constant delay case. It is natural to ask if the present method in [15] can be extended to time-varying delay case. This motivates our study in this work.

This paper aims to develop effective method for testing the admissibility for a class of singular systems with time-varying delays, which has not been fully investigated. The neutral system approach developed in [15] will be applied with modifications. However, more difficulties have to be tackled in this extension work. In order to get around these difficulties, an effective solution is proposed by embedding the time-varying parameters $h(t)$ and

* Corresponding author.
E-mail address: linchong_2004@hotmail.com (C. Lin).

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$\dot{h}(t)$ into a polytopic uncertain set in which the vertices can be calculated by setting the parameter to either lower or upper bounds [17]. Then, a delay-derivative-dependent sufficient condition in terms of a set of LMIs is proposed via the new type of LKFs, Writinger-based inequality and improved reciprocally convex combination methods. The result can also be used in testing stability of both retarded and neutral systems with time-varying delays. Several numerical examples are included to illustrate the effectiveness of the proposed method.

Notation: In this paper, $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space; $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices; the superscript 'T' denotes the matrix transportation, and for any square matrix $A\in\mathbb{R}^{n\times m}$, we define $Sym\{A\}=A^T+A$; $diag\{\cdots\}$ denotes the block diagonal matrix; $P>0$ $(P\ge 0)$ means that P is a real, symmetric and positive definite (positive semi-definite) matrix; the symbol '$\star$' represents the symmetric elements in a symmetric matrix; I_n and 0_n are, respectively, the $n\times n$ dimensional identity matrix and the $n\times n$ dimensional zero matrix. Whereas no confusion is caused, we also use I and 0 to denote, respectively, the identity matrix and the zero matrix with compatible dimensions.

2. Problem formulation and preliminaries

Consider the following linear singular system with time-varying delays:

$$E\dot{x}(t)=Ax(t)+A_dx(t-h(t)),$$

$$x(t)=\phi(t),\quad t\in[-h_2,0], \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state. E, A, $A_d\in\mathbb{R}^{n\times n}$ are known real constant matrices where E may be singular, we shall assume that rank $E=r\le n$. $\phi(t)$ is a continuously differentiable vector-valued initial function and $h(t)$ is a time-varying differentiable function satisfying

$$0\le h_1\le h(t)\le h_2,\quad d_1\le\dot{h}(t)\le d_2,\ \forall t\ge 0, \tag{2}$$

where h_1, h_2, d_1 and d_2 are given bounds.

In this paper, we aim at establishing a delay-dependent admissibility criterion, which can produce allowable upper bounds on delay as large as possible. Inspired by the neutral system approach in [15], we try to transform system (1) into a neutral form under certain constraint. To proceed, the following definitions and lemmas are needed.

Definition 1 (*Dai [2]*). The pair (E,A) is said to be regular if det $(sE-A)$ is not identically zero; the pair (E,A) is said to be impulse free if $\deg(\det(sE-A))=\mathrm{rank}(E)$.

Lemma 1 (*Xu et al. [4]*). *If the pair (E,A) is regular and impulse free, then the solution to the singular time-delay system (1) exists and is impulse free and unique on* $[0,\infty)$.

By Lemma 1, the following definition is naturally introduced.

Definition 2 (*Xu et al. [4,5]*). The singular time-delay system (1) is said to be regular, impulse free, if the pair (E,A) is regular, impulse free; the singular time-delay system (1) is said to be admissible, if it is regular, impulse free and stable.

Lemma 2 (*[2]*). *If the pair (E,A) is regular and impulse free, there exist two invertible matrices $M\in\mathbb{R}^{n\times n}$ and $N\in\mathbb{R}^{n\times n}$ such that*

$$MEN=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix},\quad MAN=\begin{bmatrix} A_1 & 0\\ 0 & I_{n-r}\end{bmatrix}. \tag{3}$$

According to Lemma 2, if the pair (E,A) is regular and impulse free, invertible matrices $M,N\in\mathbb{R}^{n\times n}$ can always be found such that

$$\overline{E}:=MEN=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix},\quad \overline{A}:=MAN=\begin{bmatrix} A_1 & 0\\ 0 & I_{n-r}\end{bmatrix}. \tag{4}$$

Let

$$\overline{A}_d=MA_dN=\begin{bmatrix} A_{d1} & A_{d2}\\ A_{d3} & A_{d4}\end{bmatrix},\quad \mu(t)=N^{-1}x(t)=\begin{bmatrix}\mu_1(t)\\ \mu_2(t)\end{bmatrix}, \tag{5}$$

where the partitions are compatible with the structure of $\overline{E}$. The system (1) is thus equivalent to

$$\overline{E}\dot{\mu}(t)=\overline{A}\mu(t)+\overline{A}_d\mu(t-h(t)), \tag{6}$$

which is also the form of

$$\dot{\mu}_1(t)=A_1\mu_1(t)+A_{d1}\mu_1(t-h(t))+A_{d2}\mu_2(t-h(t)), \tag{7}$$

$$0=\mu_2(t)+A_{d3}\mu_1(t-h(t))+A_{d4}\mu_2(t-h(t)). \tag{8}$$

Then we rewrite the second equation, by differentiating (8), as

$$\dot{\mu}_2(t)+(1-\dot{h}(t))A_{d3}\dot{\mu}_1(t-h(t))+(1-\dot{h}(t))A_{d4}\dot{\mu}_2(t-h(t))=0. \tag{9}$$

Combining (7)–(9), we have

$$\begin{bmatrix}\dot{\mu}_1(t)\\ \dot{\mu}_2(t)\end{bmatrix}=\begin{bmatrix}A_1\mu_1(t)+A_{d1}\mu_1(t-h(t))+A_{d2}\mu_2(t-h(t))\\ -\mu_2(t)-A_{d3}\mu_1(t-h(t))-A_{d4}\mu_2(t-h(t))\end{bmatrix}+(1-\dot{h}(t))\begin{bmatrix}0 & 0\\ -A_{d3} & -A_{d4}\end{bmatrix}\begin{bmatrix}\dot{\mu}_1(t-h(t))\\ \dot{\mu}_2(t-h(t))\end{bmatrix}. \tag{10}$$

Let

$$\hat{A}=\begin{bmatrix}A_1 & 0\\ 0 & -I_{n-r}\end{bmatrix},\quad \hat{A}_d=\begin{bmatrix}A_{d1} & A_{d2}\\ -A_{d3} & -A_{d4}\end{bmatrix},$$

$$C=\begin{bmatrix}0 & 0\\ -A_{d3} & -A_{d4}\end{bmatrix},\quad \hat{C}(t)=(1-\dot{h}(t))C. \tag{11}$$

Then the system (10) is of the form of a neutral type system,

$$\dot{\mu}(t)-\hat{C}(t)\dot{\mu}(t-h(t))=\hat{A}\mu(t)+\hat{A}_d\mu(t-h(t)),$$
$$\mu(t)=\varphi(t),\quad t\in[-h_2,0]. \tag{12}$$

Note that systems (12) and (1) are not equivalent, but the asymptotic stability of (12) will guarantee the admissibility of (1), and vice versa [15]. With respected to the stability analysis of system (12), we assume that all the eigenvalues of matrix $\hat{C}(t)$ are inside the unit circle, i.e., $\rho(\hat{C}(t))=\max\{|(1-d_1)\rho(C))|,|(1-d_2)\rho(C))|\}<1$, where the symbol ρ denotes the spectral radius of the matrix. So the following assumption is made.

Assumption 1. The pair (E,A) is regular and impulse free and all the eigenvalues of matrix $\hat{C}(t)$ are inside the unit circle, i.e., $\max\{|1-d_1|,|1-d_2|\}\rho(C)<1$.

Lemma 3 (*Seuret and Gouaisbaut [21]*). *For any constant matrix $R\in\mathbb{R}^{n\times n}$, $R=R^T\ge 0$, the following inequality holds for all con-*