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### **Research Article**

# Two-degree-of-freedom fractional order-PID controllers design for fractional order processes with dead-time

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#### ABSTRACT

Recently, fractional order (FO) processes with dead-time have attracted more and more attention of many researchers in control field, but FO-PID controllers design techniques available for the FO processes with dead-time suffer from lack of direct systematic approaches. In this paper, a simple design and parameters tuning approach of two-degree-of-freedom (2-DOF) FO-PID controller based on internal model control (IMC) is proposed for FO processes with dead-time, conventional one-degree-of-freedom control exhibited the shortcoming of coupling of robustness and dynamic response performance. 2-DOF control can overcome the above weakness which means it realizes decoupling of robustness and dynamic performance from each other. The adjustable parameter  $\eta_2$  of FO-PID controller is directly related to the robustness of closed-loop system, and the analytical expression is given between the maximum sensitivity specification  $M_s$  and parameters  $\eta_2$ . In addition, according to the dynamic performance requirement of the practical system, the parameters  $\eta_1$  can also be selected easily. By approximating the dead-time term of the process model with the first-order Padé or Taylor series, the expressions for 2-DOF FO-PID controller parameters are derived for three classes of FO processes with dead-time. Moreover, compared with other methods, the proposed method is simple and easy to implement. Finally, the simulation results are given to illustrate the effectiveness of this method.

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#### 1. Introduction

Nowadays, many natural and artificial systems have revealed the inherent dynamic behavior of fractional order, considerable attention has been paid to FO systems whose models are described by FO differential equations. Although a few natural phenomena can be modeled or fitted in this way, FO model can provide a reliable modeling tool in describing many real dynamic processes. Many studies have shown that FO differential equation could provide a more accurate description of many real complex physical systems than conventional integer order equations [1–5]. With further researches on fractional calculus theory in recent years, more and more mathematical models of practical systems have been gradually established by FO differential equation, such as heating-furnace [6], heat diffusion [7,8], electrical circuits [9], gas turbine [10] and water distribution in a main irrigation canal pool [11] obtained by physical properties or system identification methods. However, in actual natural and engineering systems, dead-time is widespread and often affects the performance of the

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control system, making the stability of the control system decrease, or even causing instability of closed-loop system. Thus, research on control for the process with dead-time has more great theoretical and practical significance.

In recent years, since Podlubny I [12] proposed a generalization of the traditional PID controller based on fractional order calculus, namely  $PI^{\lambda}D^{\mu}$  controller, and the integrator of order  $\lambda$  and differentiator of order  $\mu$  may be assumed real non-integer values. It has been shown the  $PI^{\lambda}D^{\mu}$  controller has extra degrees-of-freedom introduced by  $\lambda$  and  $\mu$ , which can provide better performance of the system and robustness than classic integer order PID controller, so the design methods of FO-PID controller have attracted more and more attention in the control field. Various methods have been devised for designing and tuning of FO-PID controller. For example, papers [13–15] proposed a tuning method of FO-PD and FO-PI controllers respectively, these authors also have demonstrated better properties of these types of controllers than the classical integer order PID controllers, the method results in desired gain crossover frequency and phase margin, and then the phase Bode plot at the crossover frequency is flat, the tuning method can be used by means of solving three nonlinear equations. Ziegler-Nichols (Z-N) tuning rule has been developed for FO processes with dead-time [16]. A solution to the problem of

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stabilizing the given fractional order processes was by mapping into the global stability region of the parameters for FO-PID controllers in the space which are presented in [17-20]. The main contributions of [21,22] were studies of design method in  $\text{FO-PD}^{\mu}$ and FO-PI<sup> $\lambda$ </sup> controllers based on  $H_{\infty}$  performance for FO processes with dead-time, and the stabilizing region of the parameters could be calculated via a range of frequencies respectively, which satisfied the  $H_{\infty}$ -norm constraint of complementary sensitivity function and defined the  $H_{\infty}$  boundary curve. Meanwhile, many intelligent optimization tuning methods of FO-PID controllers based on improved reduced FO processes with dead-time were argued in [23–26] by optimizing the integrated absolute error criterion or minimizing time domain performance index of the control system that were given respectively. A design technique of the hybrid fuzzy FO-PID controller was investigated for FO process with dead-time by minimizing the integrated absolute criterion and deviation in control signal [27,28]. In [29–31] a new approach was developed to design simple FO-PID controllers based on internal model control paradigm for FO processes. In many of these above studies, these focuses have been made to design and tune FO-PID controllers for FO processes directly or to improve reduced FO processes with dead-time by enhancing dynamic performance and robustness of the closed-loop control system.

In this paper, we focus on FO processes with dead-time controlled by 2-DOF FO-PID controllers, but a new approach will be investigated. 2-DOF FO-PID control structure based on the principle of IMC is proposed, which can make the robustness and dynamic performance of the system decoupled from each other, the adjustable parameter  $\eta_2$  of the FO controller is directly related to the robustness of the closed-loop system, and the analytical expressions is given between  $M_s$  and the parameter  $\eta_2$  of FO-PID controller. According to the dynamic performance requirement of practical system, parameters  $\eta_1$  can also be selected easily. And different types of 2-DOF FO-PID controllers can be obtained for three classes of FO processes with dead-time by approximating the dead-time term of processes models with the first-order Padé or Taylor series. So, 2-DOF FO-PID controller parameters tuning can rely on the performance of the set-point tracking or robustness independently and monotonously, the shortcoming of the conventional one degree of freedom IMC could be avoided. The simulation results show that the proposed method is effective.

#### 2. Preliminary

#### 2.1. Fractional order calculus

Fractional calculus is a generalization of the integration and differentiation to the non-integer (fractional) order fundamental operator  ${}_{a}D_{t}^{v}f(t)$ . A commonly used definition of the FO calculus is the Riemann–Liouville (RL) and Grünwald–Letnikov (GL) definition. If the function f(t) is continuous and differentiable in the interval [a, t]. According to RL definition, the  $\gamma$ -th order integration of a function f(t) can be expressed as

$${}^{\mathrm{RL}}_{a}D_{t}^{\gamma}f(t) = \frac{1}{\Gamma(-\gamma)} \int_{a}^{t} \frac{f(\tau)}{(t-\tau)^{\gamma+1}} d\tau, \ \gamma < 0$$

$$\tag{1}$$

and similarly the  $\gamma$ -th order differentiation of the function f(t) is defined as

$${}^{\mathrm{RL}}_{a}D_{t}^{\gamma}f(t) = \frac{1}{\Gamma(n-\gamma)}\frac{d^{n}}{dt^{n}}\int_{a}^{t}\frac{f(\tau)}{(t-\tau)^{\gamma-n+1}}d\tau, \quad \gamma > 0$$
<sup>(2)</sup>

For  $n - 1 < \gamma < n$ ,  $\Gamma(\cdot)$  is Euler's gamma function, *a* and *t* are the limits and  $\gamma$  is the order of the operation.

And an alternative definition based on the concept of fractional differentiation is GL definition given by

$${}_{a}^{GL}D_{t}^{\gamma}f(t) = \lim_{h \to 0} \frac{1}{h^{\gamma}} \sum_{k=0}^{\left[(t-a)/h\right]} (-1)^{k} {\gamma \choose k} f(t-kh)$$
(3)

where  $[\cdot]$  means the integer part, h represents sampling period and

$$\binom{\gamma}{k} = \frac{\gamma(\gamma-1)(\gamma-2)\dots(\gamma-k+1)}{k!}$$
(4)

The Laplace transform form of a  $\gamma$ -th derivative operator with  $\gamma \in R_+$  of a signal f(t) is given by

$$L\{_{a}D_{t}^{\gamma}f(t)\} = s^{\gamma}L\{f(t)\} - \sum_{i=0}^{k-1} s^{\gamma-1-i}{}_{a}D_{t}^{\gamma}f(0)$$
(5)

where  $k - 1 < \gamma < k$  and  $k \in N$ , which in zero initial conditions can be reduced to

$$L\left\{{}_{a}D_{t}^{\gamma}f(t)\right\} = s^{\gamma}F(s) \tag{6}$$

In the present simulation study, to implement the FO operator  $s^{\gamma}$ , it is employed by oustaloup recursive algorithm [32] for the approximation. If the expected frequency range is selected as  $[\omega_a, \omega_b]$ , to acquire a good accuracy in this frequency range, the approximate transfer function of a continuous FO operators  $s^{\gamma}$  via integer order components with oustaloup recursive algorithm is calculated as follows

$$G_f(s) = s^{\gamma} \approx K \prod_{j=-N}^{N} \frac{s + w_j}{s + w_j^*}, \quad (0 < \gamma < 1)$$
 (7)

where the zeros, poles and gain K can be evaluated respectively from

$$K = \left(\frac{\omega_b}{\omega_a}\right)^{-\frac{j}{2}} \prod_{j=-N}^{N} \frac{\omega_j'}{\omega_j}$$
  

$$\omega_j = \omega_a \left(\frac{\omega_b}{\omega_a}\right)^{\frac{k+N+0.5(1-\gamma)}{2N+1}}$$
  

$$\omega_j' = \omega_a \left(\frac{\omega_b}{\omega_a}\right)^{\frac{k+N+0.5(1-\gamma)}{2N+1}}$$
(8)

where  $\gamma$  is the order of the differ-integration and (2N+1) is order of the filter. In our simulation, for the approximation of FO differentiator, frequency range of practical interest is set to be from 0.001 Hz to 1000 Hz, and *N* is selected as 4 for the proper accuracy of the approximation. If  $\gamma > 1$ , FO derivative can be written as  $s^{\gamma} = s$ .  $s^{\gamma-1}$ , where only fractional part is approximated via Eq. (7).

#### 2.2. Fractional order PID controller

The classical Proportional–Integral–Derivative (PID) controller is still widely recognized as one of the simplest yet most effective control strategies in the control industry. Based on the principles of fractional calculus, the following four types of FO-controllers have been briefly introduced in [33], namely, CRONE controller, FO-lead-lag compensator, FO-TID (Tilt-Integral-Derivative) controller, and  $\text{PI}^{\lambda}D^{\mu}$  controller. For example, fractional order TID controller, which has structure similar to a traditional integer order PID controller but the proportional component is replaced with a tilted component having a transfer function *s* to the power of (-1/n). The resulting transfer function of TID controller has the form

$$C(s) = \frac{T_i}{s^{1/n}} + \frac{K_i}{s} + K_d s, (n > 0)$$
(9)

where  $T_i$ ,  $K_i$ , and  $K_d$  are the controller constants, and n is a nonzero real number, preferably  $n \in (2, 3)$ .

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