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Research Article

A single sensor and single actuator approach to performance tailoring over a prescribed frequency band

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ABSTRACT

Restricted sensing and actuation control represents an important area of research that has been overlooked in most of the design methodologies. In many practical control engineering problems, it is necessitated to implement the design through a single sensor and single actuator for multivariate performance variables. In this paper, a novel approach is proposed for the solution to the single sensor and single actuator control problem where performance over any prescribed frequency band can also be tailored. The results are obtained for the broad band control design based on the formulation for discrete frequency control. It is shown that the single sensor and single actuator control problem over a frequency band can be cast into a Nevanlinna–Pick interpolation problem. An optimal controller can then be obtained via the convex optimization over LMIs. Even remarkable is that robustness issues can also be tackled in this framework. A numerical example is provided for the broad band attenuation of rotor blade vibration to illustrate the proposed design procedures.

1. Introduction

Control systems are always restricted in one form or another, e.g. in restricted structure control (RSC) [1–3], controller structure may be specified to be of reduced order or PID etc.; in prescribed performance control (PPC), it is the control performance that is prescribed a priory in terms of certain evaluation index, e.g. transient and steady-state error [4], bounding error under disturbances [5,6] or stability margins [7]. There are also restricted system signals due to the fact that any realistic signal for system inputs, system outputs, or system states must be bounded in magnitude or norm, e.g. actuators will eventually saturate and sensors can only work over limited operational range etc. In restricted manifold control, however, the control system is defined on a fibered manifold, and it concerns the problem of restricting control systems to a submanifold of the state space [8,9]. There are of course many other restrictions enforced from product requirements such as weight, size, cost, even reliability and life span etc.

There is yet another type of restriction that can be named *restricted sensing and actuation control*. This is different from restricted system signal discussed above. In restricted system signal control, it is primarily the saturation problem that should be tackled; however, in restricted sensing and actuation control, it is the lack of number of sensors and/or actuators that should be concerned. This issue has not been generally recognized, since most of the control design methods make the implicit assumption that sensors or actuators can be available or located in the region where control action is required. However, this may be either not feasible or prohibitively expensive for many practical engineering systems. As a consequence, the optimum solution is often based solely on information local to the actuators and implementation can result in deteriorated performance at other locations. Such problems are particularly evident in large scale interconnected structures where it is neither feasible nor cost effective to provide a large distribution of sensors and actuators. Attainment of a globally optimal solution may therefore necessitate the implementation of a locally sub-optimal one.

Indeed, the restricted sensing and actuation control problem is solved in its extreme case, namely single sensor and single actuator control (SSSAC) in reference [10] for nonlinear systems. In this paper, the SSSAC problem for linear systems is considered. However a fundamentally different approach is taken for handling the linear counterpart, and in specific, the method will allow disturbance attenuation *over any specified frequency band*. To further appreciate the importance of the problem under discussion, consider the case in servo systems where the frequency band of the disturbance is known (but the model of the disturbance is not necessarily known or even observable), conventional approaches such as internal model control [11], disturbance observer (see [12–

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14] for example), adaptive disturbance rejection control [15] or even H_2 control methods can be used for attenuating the unknown disturbance. However, these methods are not taking fully advantage of the known frequency band of the disturbance signal. The same situation can be compared with those methods for underactuated mechanical systems control [16] and sensorless control [17,18], where there are fewer independent control actuators than degrees of freedom to be controlled. In fact, it can even be desirable to design a controller that can provide optimal attenuation of the disturbance over the targeted frequency band, even with the sacrifice of performance outside that frequency band. Such consideration can be very important for the case where there is no disturbance to be rejected outside a target frequency band, e.g. the disturbance is not known but its power spectrum is within a frequency band of $[\omega_1 \quad \omega_N]$ rad/s, we may thus design a controller with optimal disturbance attenuation over $[\omega_1 \quad \omega_N]$ rad/s, while accepting performance deterioration outside the frequency band (due to the Bode's integral relationships encoding the fundamental design limitations). Thus as long as only the performance within $[\omega_1 \quad \omega_N]$ rad/s is concerned, a design methodology is necessiated with the systematic shaping of the closed loop performance soly within $[\omega_1 \quad \omega_N]$ rad/s.

To summarize, the proposed method shall target the following two problems simultaneously: (1) *using only one sensor and one actuator for controlling multivariable systems*; (2) *attenuating disturbance over any prescribed frequency band*. The paper is structured as follows: Section 2 formulates the problem to be tackled; Section 3 provides solutions and Section 4 considers the controller implementation issue, while a numerical example is given to validate the proposed method in Section 5. Finally Section 6 concludes the paper.

2. Problem formulation

2.1. Preliminaries

Consider the following single sensor, single actuator control system with multiple exogenous disturbances:

$$\begin{bmatrix} y(s) \\ z_1(s) \\ z_2(s) \\ \vdots \\ z_n(s) \end{bmatrix} = \begin{bmatrix} G_{00}(s) & G_{01}(s) & \cdots & G_{0n}(s) \\ G_{10}(s) & G_{11}(s) & \cdots & G_{1n}(s) \\ G_{20}(s) & G_{21}(s) & \cdots & G_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{n0}(s) & G_{n1}(s) & \cdots & G_{nn}(s) \end{bmatrix} \begin{bmatrix} u(s) \\ d_1(s) \\ d_2(s) \\ \vdots \\ d_n(s) \end{bmatrix} \tag{1}$$

In the above equation, $u(s)$ represents a scalar control input and $d_i(s)$ the ith exogenous disturbance. $y(s)$ is the only available scalar variable for feedback while $z_i(s)$ for $i=1,\cdots,n$ are the performance variables to be controlled but unavailable for feedback. To simplify the notations and better explain the key point of the following solutions, the above representation is reduced to the following two-input, two-output system:

$$\begin{bmatrix} y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u(s) \\ d(s) \end{bmatrix} \tag{2}$$

where all the terms are now scalars. Rewrite Eq. (2) in its frequency response function form:

$$\begin{bmatrix} y(j\omega) \\ z(j\omega) \end{bmatrix} = \begin{bmatrix} g_{11}(j\omega) & g_{12}(j\omega) \\ g_{21}(j\omega) & g_{22}(j\omega) \end{bmatrix} \begin{bmatrix} u(j\omega) \\ d(j\omega) \end{bmatrix} \tag{3}$$

The single sensor and single actuator control problem is to use only the feedback action $u(j\omega) = -k(j\omega)y(j\omega)$ to control the whole system, that is, attenuation of both $y(j\omega)$ and $z(j\omega)$ is achieved over a (any) prescribed frequency band $\omega \in [\omega_1 \quad \omega_N]$ rad/s. From Eq. (3), attenuation of both $y(j\omega)$ and $z(j\omega)$ at a frequency ω is equivalent to the following two conditions:

$$|y(j\omega)| = \left|[1+g_{11}(j\omega)k(j\omega)]^{-1}g_{12}(j\omega)d(j\omega)\right| \le |g_{12}(j\omega)d(j\omega)| \tag{4}$$

$$|z(j\omega)| = \left|\left\{-g_{21}(j\omega)k(j\omega)[1+g_{11}(j\omega)k(j\omega)]^{-1}g_{12}(j\omega) + g_{22}(j\omega)\right\}d(j\omega)\right| \le |g_{22}(j\omega)d(j\omega)| \tag{5}$$

where $|\bullet|$ operation takes the magnitude of the frequency response function. For a particular discrete frequency ω_0, the above two conditions can be rewritten into

$$\left|[1+g_{11}(j\omega_0)k(j\omega_0)]^{-1}\right| \le 1 \tag{6}$$

$$\left|1 - \frac{g_{21}(j\omega_0)k(j\omega_0)[1+g_{11}(j\omega_0)k(j\omega_0)]^{-1}g_{12}(j\omega_0)}{g_{22}(j\omega_0)}\right| \le 1 \tag{7}$$

At first sight, it may be suspected that the problem can be solved using conventional loop shaping method, since it is not difficult to choose a controller $k(j\omega_0)$ such that the transmission from the exogenous disturbance $d(j\omega_0)$ to either $y(j\omega_0)$ or $z(j\omega_0)$ is attenuated. Indeed, the former is a sensitivity shaping problem while the latter can be approached using $H\infty$ optimization. However it turns out that conditions (6) and (7) may not be satisfied simultaneously: optimization of attenuation of $y(j\omega_0)$ often leads to substantial enhancement of $z(j\omega_0)$, and vice versa. Simultaneous shaping of both $y(j\omega_0)$ and $z(j\omega_0)$ is a challenging problem, particularly under a specified frequency band $[\omega_1 \quad \omega_N]$.

2.2. Formulate SSSAC problem over prescribed frequency band

The above interesting problem can be stated as achieving reductions in both $y(j\omega)$ and $z(j\omega)$ (where possible) over a frequency band $[\omega_1, \omega_N]$ using a single sensor and a single actuator. It is formulated mathematically as finding a control law $u(j\omega) = -k(j\omega)y(j\omega)$ such that conditions (6) and (7) are satisfied simultaneously over a frequency band $[\omega_1, \omega_N]$. To proceed, first define the sensitivity at a discrete frequency $\omega = \omega_0$ by

$$S(j\omega_0) = (1+g_{11}(j\omega_0)k(j\omega_0))^{-1} \tag{8}$$

and let

$$\alpha(j\omega_0) = S(j\omega_0) - 1 \tag{9}$$

then reduction in $y(j\omega)$ for the discrete frequency $\omega = \omega_0$ is equivalent to the condition:

$$|\alpha(j\omega_0)+1| < 1 \tag{10}$$

Further define:

$$\beta(j\omega_0) = \alpha(j\omega_0)g^{-1}(j\omega_0) \tag{11}$$

$$g(j\omega_0) = \frac{g_{11}(j\omega_0)g_{22}(j\omega_0)}{g_{12}(j\omega_0)g_{21}(j\omega_0)} \tag{12}$$

then it can be seen that reduction in $z(j\omega)$ for the discrete frequency $\omega = \omega_0$ is equivalent to the condition:

$$|\beta(j\omega_0)+1| < 1 \tag{13}$$

Noting that Eq. (11) is a Möbius transformation, it is concluded that simultaneous reduction of $y(j\omega)$and $z(j\omega)$ is achievable for this discrete frequency $\omega = \omega_0$ if and only if *the mapping of the unit circle* (and its interior) $|\beta(j\omega)+1| < 1$ on the complex *α−plane intersects the unit α-circle* (and its interior) $|\alpha(j\omega)+1| < 1$, and the corresponding reduction is the scaling with respect to each circle. It is noted that *the β-circle* always intersects unit *α-circle* at the origin (0, 0). At this point, the performance responses of y and z are simply their open loop responses with controller $k=0$. Consequently this implies that the two circles will intersect for controllers other than $k=0$, henceforth implying that system