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Research article

Robust observer-based passive control for uncertain singular time-delay systems subject to actuator saturation ☆

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ABSTRACT

This paper investigates the problem of robust observer-based passive control for uncertain singular time-delay system subject to actuator saturation. A polytopic approach is used to describe the saturation behavior. First, by constructing Lyapunov-Krasovskii functional, a less conservative sufficient condition is obtained which guarantees that the closed-loop system is regular, impulse free, stable and robust strictly passive. Then, with this condition, the design method of state feedback controller and the observer are given by solving linear matrix inequalities. In addition, a domain of attraction in which the admissible initial states are ensured to converge asymptotically to the origin is solved as a convex optimization problem. Finally, some simulations are provided to demonstrate the effectiveness and superiority of the proposed method.

1. Introduction

Singular systems, which are also referred to as descriptor systems, implicit systems or generalized state-space systems. Over the past decades, it has extensive applications in many areas such as electrical networks, circuit systems, power systems, economic systems, robotic systems, chemical systems and other areas [1,2], due to the fact that singular systems can better describe the behavior of some physical systems than regular ones standard state-space systems [3,4]. Hence a great number of fundamental nations and theories for singular systems have been studied, such as stability, stabilization, H_∞ control problem [5–9] and so on. Moreover, the time delay singular representation, which is a mixture of algebraic and differential equations with time delay, often encounters in various practical systems [10,11]. And the fact that time-delay is a major cause of instability and poor performance of dynamic systems. As a consequence, the study of time delay singular systems has been attracted much attention in recent years [12–16].

On the other hand, significant attention has been taken to passivity theory, which is one of the most useful forms of dissipativity theory and plays an important role in the analysis and design of linear and nonlinear systems [17–23]. In [18], the problem of robust passive control is considered for uncertain singular time-delay systems, and three types of controllers are discussed, namely, state feedback controller, observer-based state feedback controller and dynamic output feedback controller. However, the delay-independent approach was applied in [18] to obtain the results, which have been proved to be more conservative than the ones with the delay-dependent approach. [21] discussed a class of finite-time robust passive control for a class of uncertain Lipschitz nonlinear systems with time-delays, but it just applies to non-singular systems.

In another aspect, all physical systems are subject to saturation constraints, so we should impose additional constraints on the analysis and design. Various control problems for systems with actuator saturation have been extensively studied in the literature [24–30]. The control synthesis problem for a class of linear time-delay systems with actuator saturation is investigated in [24]. [28,29] studied fuzzy adaptive output feedback control design for uncertain nonlinear systems with input saturation, but it didn't apply to singular system. And in [24] the problem of robust H_∞ control for a nonlinear singular time-delay system with saturation actuators. However, it without considering uncertainty. As we know, the research on singular systems has been achieved a lot of great results. However, few works have been dealt with the problem of singular systems in the presence of actuator saturation, see for example [24–27]. It established a set of conditions under which an ellipsoid is contractively invariant with respect to a singular linear system under a saturated linear feedback in [26,27,30].

The last but not least, it has been recognized that in most practical situations, state variables are generally not easily available through output measurement. Therefore, the controller design that does not require complete access to the state vector is preferable. That is why many observer design

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problems, which are concerned with using the available information on inputs and outputs to reconstruct the unmeasured states of the studied system, have been investigated for many practical applications in [31–39]. For instance, the authors in [31,35–37,39] investigated the problem of observer-based control for nonsingular systems. Meanwhile, more and more references have been carried out on singular systems [32–34,38]. But yet, Ma et al. design a state observer for a class of nonlinear descriptor systems without considering time delay [32], Kao et al. develop a Observer-based H_∞ sliding mode controller design for uncertain stochastic singular time-delay systems without considering the input saturation [33]. And in [34] studied the observer-based stabilization and control design for uncertain singular systems with time-delay, while it haven't analyze the passivity of it. Therefore, to the best of our knowledge, the problem of robust observer-based passive control for uncertain singular time-delay systems subject to actuator saturation has not been investigated in available literatures. And that is the main purpose of our study.

The aim of this paper is to design an observer-based state feedback controller such that the uncertain singular time-delay systems is not only robustly stable, but also satisfies robust strictly passive. A delay-dependent LMI condition which guarantees that the uncertain singular time-delay systems are admissible is derived. And introducing a lemma which plays a significant role in the derivation of a less conservative delay-dependent result. Under the conditions, the estimation of stability region, the observer and the controller with state feedback are given by solving a convex optimization problem. Finally some numerical examples are provided to demonstrate the efficiency of the proposed results. The main structure of the paper is arranged as follows: In Section 2, a new uncertain singular time-delay system subject to actuator saturation and a last state observer are constructed, and reviews some necessary definitions and lemmas. In Section 3, first, some delay-dependent stability conditions are established. then, a observer-based state feedback controller is designed. Three numerical examples and a equivalent circuit of transistor network system are given in Section 4 to demonstrate the effectiveness and applicability of our theoretical findings. Finally, in Section 5, the conclusion of the paper is summarized.

Notation: Throughout this article, For $A \in R^{n\times n}$, A^{-1} and $A^{-\mathrm{T}}$ denote the inverse and the transpose of A, respectively. Symmetric elements in the matrix are denoted by $*$. $sym(X)$ stands for $X + X^{\mathrm{T}}$. R_n and $R_{n\times m}$ denote respectively the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. I and 0 denote the identity matrix and the zero matrix of appropriate dimensions respectively. For example, $0_{n\times m}$ denotes the $n \times m$ zero matrix. And $diag\{\cdots\}$ represents a block-diagonal matrix.

2. Description of the problem and main results

Consider a uncertain singular time-delay system subject to actuator saturation and norm-bounded uncertainties as follow:

$$\begin{cases} E\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)) + \\ \qquad (B + \Delta B)\mathrm{sat}(u(t)) + (B_\omega + \Delta B_\omega)\omega(t) \\ y(t) = C_1 x(t) \\ z(t) = (C + \Delta C)x(t) + (C_\omega + \Delta C_\omega)\omega(t) \\ x(t) = \phi(t),\ t \in [-d, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $\omega(t) \in \mathbb{R}^q$ is the disturbance input which belongs to

$$w_\eta = \left\{ \omega(t):\ \int_0^\infty \omega^{\mathrm{T}}(t)\omega(t)dt < \eta,\ \eta > 0 \right\},$$

$z(t) \in \mathbb{R}^m$ is the control output, $d(t)$ is a time-varying continuous function that satisfies $0 \le d(t) \le d$ and $\dot{d}(t) \le \mu$. $y(t) \in \mathbb{R}^l$ is the measured output, $\phi(t)$ is a compatible vector valued initial condition. The saturating term $\mathrm{sat}(u(t))$ in (1) is a vector-valued function defined as follows

$$\mathrm{sat}(u(t)) = \left[\mathrm{sat}(u_1(t)),\ \ldots,\ \mathrm{sat}(u_p(t)) \right]^{\mathrm{T}},$$

where without loss of generality,

$$\mathrm{sat}(u_i(t)) = \mathrm{sign}(u_i(t))\min\left\{1,\ |u_i(t)|\right\}.$$

The matrix $E \in \mathbb{R}^{n\times n}$ is singular with $\mathrm{rank}(E) = r \le n$, A, A_d, B, B_ω, C, C_ω and C_1 are known real constant matrices of appropriate dimensions, we assume that C_1 is of full-row rank. ΔA, ΔA_d, ΔB, ΔB_ω, ΔC_ω and ΔC are unknown matrices representing norm-bounded parametric uncertainties and are assumed to be certain bound compact sets and of the form:

$$\begin{aligned} &\Delta A = E_1F_1(t)H_1, \quad \Delta A_d = E_2F_2(t)H_2,\ \ \Delta B = E_3F_3(t)H_3, \\ &\Delta B_\omega = E_4F_4(t)H_4,\ \Delta C = E_5F_5(t)H_5, \quad \Delta C_\omega = E_6F_6(t)H_6, \end{aligned} \tag{2}$$

where E_i, H_i, $(i = 1, \ldots, 6)$ are known real constant matrices with appropriate dimensions, and $F_i(t)$, $(i = 1, \ldots, 6)$ are unknown real and possibly time-varying matrices satisfying

$$F_i^{\mathrm{T}}(t)F_i(t) \le I,\ (i = 1, \ldots, 6), \tag{3}$$

Now, consider a state observer and feedback controller described by

$$\begin{cases} E\dot{\bar{x}}(t) = A_c\bar{x}(t) + A_D\bar{x}(t - d(t)) + B_c u(t) + L(y(t) - \bar{y}(t)) \\ \bar{y}(t) = C_1\bar{x}(t) \\ u(t) = K\bar{x}(t) \end{cases} \tag{4}$$

where $\bar{x}(t)$ is the estimated state, K and L are state feedback gain matrix and observer gain matrix to be designed, respectively. They are matrices to be determined with appropriate dimensions. Define the state estimated error $e(t) = x(t) - \bar{x}(t)$ and the error system can be written in the form as follows:

$$\begin{aligned} E\dot{e}(t) = {} & (A + \Delta A - A_c)x(t) + (A_c - LC_1)e(t) + A_D e(t - d(t)) \\ & + (A_d + \Delta A_d - A_D)x(t - d(t)) - B_c u(t) \\ & + (B + \Delta B)\mathrm{sat}(u(t)) + (B_\omega + \Delta B_\omega)\omega(t). \end{aligned} \tag{5}$$

Remark 1. The method of the observer design proposed in this paper has a broad scope of application. In the previous articles, the coefficient matrices of the observer are often the same as the original singular system, such as [21]. However, the matrices of the status and the control input for the observer constructed by us are inconsistent with the original system and even unknown. This makes our proposed method has a wider practical application.

Definition 1 (*[1]*).

(1). The singular time-delay system

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t - d(t)) \\ x(t) = \phi(t),\ t \in [-d, 0] \end{cases} \tag{6}$$

is said to be regular and impulse free, if the pair (E, A) and $(E, A + A_d)$ are regular and impulse free.