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Research article

Augmented nonlinear differentiator design and application to nonlinear uncertain systems

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ABSTRACT

In this paper, an augmented nonlinear differentiator (AND) based on sigmoid function is developed to calculate the noise-less time derivative under noisy measurement condition. The essential philosophy of proposed AND in achieving high attenuation of noise effect is established by expanding the signal dynamics with extra state variable representing the integrated noisy measurement, then with the integral of measurement as input, the augmented differentiator is formulated to improve the estimation quality. The prominent advantages of the present differentiation technique are: (i) better noise suppression ability can be achieved without appreciable delay; (ii) the improved methodology can be readily extended to construct augmented high-order differentiator to obtain multiple derivatives. In addition, the convergence property and robustness performance against noises are investigated via singular perturbation theory and describing function method, respectively. Also, comparison with several classical differentiators is given to illustrate the superiority of AND in noise suppression. Finally, the robust control problems of nonlinear uncertain systems, including a numerical example and a mass spring system, are addressed to demonstrate the effectiveness of AND in precisely estimating the disturbance and providing the unavailable differential estimate to implement output feedback based controller.

1. Introduction

Measured signals obtained by physical sensors are inevitably corrupted by additive noises, such as electronic shocks, hardware noises or environment noises, among others. Differentiation of a noisy signal has attracted increasing attention from research community and the industry practitioners due to their widespread applications in a variety of engineering areas [1–5]. The main design specification of differentiators is to calculate signal differentiation with adequate noise immunity level for the purposes of efficient control synthesis and implementation [6,7]. For example, an asymptotic differentiation approach of signal is incorporated with traditional feedback controller for estimating angular acceleration and achieving velocity regulation of DC motor, which greatly reduces the controller implementation cost [8]. In an analogous manner, a discrete-time variable-structure-based control strategy for permanent-magnet synchronous motor (PMSM) is developed by integrating with a speed reconstruction approach [9], where the first-order derivative of the encoder position measurement is obtained by a robust digital differentiator. In the field of navigation and control for aircrafts, carrier phase rate is determined through a nonlinear tracking differentiator (NTD) using carrier phase measurements from the Global Navigation Satellite System (GNSS) [10]. In [11], flight velocity and acceleration are successfully achieved from the position measurement by exploiting a high-order continuous nonlinear differentiator. Further extension of high-order differentiator with application to under-actuated quadrotor can be found in [12], wherein the estimation of velocity and uncertainties can be synchronously extracted using the position measurement alone. In addition to the aforementioned scenarios that differentiators are applied in feedback channel, actively eliminating control saturation and 'derivative explosion' issue existed in the traditional back-stepping control framework can both be effectively dealt with by adopting NTD in feedforward channel [13–16]. Generally speaking, if the measurement noise is sufficiently large, the corrupted differential signals used in the feedback or feedforward loops may severely destabilize the closed-loop system, thus robustness performance against noise perturbation, the convergent rate and ease of implementation are all significant factors in the assessment of differential estimation quality.

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To achieve a satisfactory differential estimation with good convergence property, many researchers have dedicated to developing various differentiators. A sliding mode technique based differentiator is proposed under the assumption that knowledges on the upper bound of the derivative of input signal is available [17]. Nevertheless, this assumption can seldom be satisfied in practice. A finite-time-convergent differentiator is developed in [18], but chattering phenomenon in differential estimation is inevitable, such disadvantage constrains its use in practical engineering. In [19], NTD using a second-order optimal acceleration function with one switching term is proposed. However, it suffers from the drawback of limited convergent speed. To improve the rapidity, since the dynamic response of TD is mainly dependent on the appropriate selection of acceleration function, pioneering contributions on constructing more efficient differentiators with various acceleration functions are extensively investigated, including power function [20], finite-time-convergent function [18] and sigmoid function [21,22]. Among which, sigmoid tracking differentiator (STD) naturally integrates the benefits of the characteristics of the existing linear and nonlinear differentiators, and possesses the property of globally fast convergent speed, the property of which will be further exploited in the paper. Furthermore, the phase lag of differentiator is addressed by the employment of feedforward action [23], in which the magnitude characteristics of resultant differentiator is close to that of the ideal differentiator, implying that the improvement in convergent speed benefiting from feedforward is achieved at the expense of sacrificing noise filtering property. Thus, it is definitely not suitable for handling the measurement noise in signal differentiation.

It should be pointed out that the existing design framework of differentiators are formulated in high-gain techniques, indicating that they may dangerously amplify the measurement noise in its differential estimates when a high gain is used, irrespective of how acceleration function is selected. In addition, since the tradeoff always exists between the speed of state recovery and the immunity to measurement noise in observer theory [24], the existing differentiators have to compromise between the noise filtering and state reconstruction speed, i.e., better noise-filtering property in signal differentiation is often achieved at the expense of noticeable time delay. Furthermore, no specific manipulation on noise elimination is taken into account in differentiator design. Therefore, the available differentiators have limited robustness against measurement noise and fall short of supplying a differential estimate with adequate noise immunity level. And how to further relax the contradictory and enhance the noise attenuation ability becomes an imperative and challenging problem, which should be addressed by exploring advanced signal differentiation approaches that will not only provide a rapid estimate, but will also guarantee smooth and noise-less property.

Motivated by the above analysis, to overcome the limitation of the existing differentiators in handling measurement noise and take advantages of STD in fast convergent rate, an augmented nonlinear differentiator (AND) based on sigmoid function is developed to calculate the noise-less time derivative under noisy measurement condition. It is worth noting that the improved mechanism not only can be confined to the designed differentiator, but also can be suitable for enhancing noise-filtering capability for the existing differentiators (including high-order differentiators), which further enriches the theoretical content of differentiators. In addition, the superiority and effectiveness of AND in significantly reducing the additive noises is demonstrated through extensive comparisons and application on handling disturbance rejection problems of nonlinear uncertain systems. The main contributions in this paper are summarized as follows.

- The idea of augmenting the signal dynamics with integrated noisy output to account for eliminating noises is firstly proposed under differentiator framework, based on which, an augmented nonlinear differentiator (AND) based on sigmoid function is developed, which can be treated as an extension of STD from theoretical structure perspective, since the noise filtering ability in differential estimate is enhanced while good convergence property is also retained. In addition, the proposed noise-tolerance technique can be easily extended to formulate the corresponding high-order differentiators to obtain multiple derivatives with improved signal-to-noise ratio, irrespective of how acceleration function is selected.
- The convergence property and robustness performance against noise are investigated via singular perturbation theory and describing function method, respectively. Also, the stability criteria is derived as the guideline for tuning parameter gains in differentiators.
- The robust control problems of nonlinear uncertain systems, including a numerical example and a mass spring system, are addressed to illustrate the effectiveness of AND in precisely estimating the disturbance and providing the unavailable differential estimate to implement output feedback based controller, which offers an efficient alternative to handle active disturbance rejection problem from signal differentiation perspective.

The paper is organized as follows. Some Preliminaries and Problem formulation are introduced in Section 2. In Section 3, main results on the proposed AND are given. In Section 4, frequency analysis is presented. Comparison and application to nonlinear uncertain systems is shown in Section 5. The paper ends with a few concluding remarks in Section 6.

2. Preliminaries and problem formulation

In this section, some fundamental theorems on designing tracking differentiator is introduced, based on which many efficient differentiators can be readily derived, including finite-time-convergent differentiator [18], sigmoid tracking differentiator [21,22], rapid-convergent nonlinear differentiator [25].

The general form of a tracking differentiator is given as below:

Lemma 1. *[26,27]: Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a locally Lipschitz continuous function with $f(0,0)=0$, the following system is globally asymptotically stable with respect to the equilibrium point $(0,0)$:*

$$\begin{cases} \dot{x}_1(t) = x_2(t),\ x_1(0) = x_{10} \\ \dot{x}_2(t) = f(x_1(t), x_2(t)),\ x_2(0) = x_{20} \end{cases} \tag{1}$$

provided that the solution of differential system (1) satisfies $(x_1(t), x_2(t)) \to 0$ as $t \to \infty$, where (x_{10}, x_{20}) is any given initial value. Then for any differential and bounded input signal $v(t)$ with the condition of $A = \sup_{t\in[0,\infty)}|\dot{v}(t)| < \infty$, the solution of the following tracking