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Dominant pole placement with fractional order PID controllers: D-decomposition approach

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ABSTRACT

Dominant pole placement is a useful technique designed to deal with the problem of controlling a high order or time-delay systems with low order controller such as the PID controller. This paper tries to solve this problem by using D-decomposition method. Straightforward analytic procedure makes this method extremely powerful and easy to apply. This technique is applicable to a wide range of transfer functions: with or without time-delay, rational and non-rational ones, and those describing distributed parameter systems. In order to control as many different processes as possible, a fractional order PID controller is introduced, as a generalization of classical PID controller. As a consequence, it provides additional parameters for better adjusting system performances. The design method presented in this paper tunes the parameters of PID and fractional PID controller in order to obtain good load disturbance response with a constraint on the maximum sensitivity and sensitivity to noise measurement. Good set point response is also one of the design goals of this technique. Numerous examples taken from the process industry are given, and D-decomposition approach is compared with other PID optimization methods to show its effectiveness.

1. Introduction

The pole placement approach is very popular technique in linear control systems because of its simple design procedure and predictable performance in closed loop systems [1]. The idea behind the pole placement approach is to place the closed loop poles at pre-determined locations in the complex s-plane [2]. This is essential since location of the poles corresponds directly to the eigenvalues of the system, which in return determine the behavior of the closed loop dynamics. However, in order to achieve arbitrary position of system poles for a given plant model, degree of the controller should be at least the plant order minus one. This is a major limitation of this technique if we have to use a low order output feedback controller for a high order, or time delay, plant. One typical example is the use of PID controllers in industrial applications [3]. Implementing pole placement procedure with PID controllers could be a difficult task due to a limited number of adjustable parameters.

It is well known that PID controller is the most common way of using feedback in engineering systems. Based on some surveys, 97% of regulatory controllers utilize PID feedback [4]. The reason for this is its simple, yet effective algorithm. However, in practice, a large number of the PID controllers are poorly tuned. Moreover, as stated in [5]: "… 25% of all PID controller loops use default factory settings, implying that they have not been tuned at all". One major reason may be that in industry exist practically an infinite number of different plant models. There is no general tuning method, and a lot of design techniques rely on restrictions on the process model. One way to cope with this is designing a more robust controller.

A fractional order $PI^{\beta}D^{\alpha}$controller [6] is a generalization of the classical PID controller, where β and α represent the fractional orders of the integral and derivative parts, respectively. In this paper, the fractional order PID^{α} controller is used $(\beta = 1)$ with only the derivative term having a fractional order. As a consequence, it gives us one additional tuning parameter of the controller, ensuring a more robust controlled system. A lot of tuning methods for FO PID controllers have been proposed in the literature. One of the tuning rules, developed for fractional PI controllers and based on the maximization of the integral gain with a constraint on robustness, is given in [7,8]. Similarly, in [9] fractional PI controller is tuned to make the system more robust to gain changes and time

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constant changes. Tuning rules for integer order and fractional order PID controllers that minimize the integrated absolute error, subjected to the constraint on maximum sensitivity, is proposed in [10]. One of the objectives of this work is to apply fractional order control to improve the system control performances by using the D-decomposition approach.

The basic idea of the D-decomposition approach can be traced back to Vishnegradsky [11] who treated the two middle coefficients of the third degree characteristic equation as parameters. In the plane of the variable coefficients, a diagram is plotted and stability domain is determined. But it was Yu. Neimark [12,13] who extended Vishnegradsky's work to higher degree systems, developed the rigorous algorithm, and coined the name of "D-decomposition". By utilizing Neimark's procedure, the mapping of the imaginary axis of the complex s-plane onto the plane of the controller parameters allows us to determine the stability region of a closed loop system in the parameter plane. Later, Mitrović[14] proposed the mapping of contours other than imaginary axis, and Šiljak[15] extended the approach to the case of nonlinear parameter dependence. Lately, Gryazina and Polyak[16] further developed the D-decomposition method and used it to estimate the number of root invariant regions.

The dominant pole placement was introduced by [17], and further developed by [18]. It relies on the fact that for even more complicated systems the response is often characterized by the dominant poles. So, the idea behind dominant pole placement design is to put two closed loop poles to be in the dominant region, and all other poles to be located outside of this region. This is accomplished by choosing the parameters of PID controller which guarantee the dominance of the assigned poles. The conventional dominant pole design used in [18] is based on a simplified model of plants, and dominance of the chosen poles cannot be guaranteed. This approach, if not well handled, could even result in instability of the closed loop system. To avoid this, guaranteed dominant pole placement with PID controllers has been proposed by Wang et al.[19]. Two methods based on the root locus and Nyquist plot have been developed to ensure the dominance of the two assigned poles. In the cases when there is no transport delay in the process model the root locus method is used, and the Nyquist stability criteria is applied to handle time delay systems. In a more recent publication by Šekara et al.[20], a generalization of the results published in [19] has been presented. It is shown that the root locus method can be utilized regardless of the transport delay in the plant model.

In the present paper, an alternative approach for dominant pole placement by using the D-decomposition method has been proposed. Both processes, with or without time delay, can be tackled by this technique in a unified manner. In fact, the proposed approach can be applied not only to time delay systems, but also to fractional and distributed parameter processes. Design procedure of D-decomposition method is straightforward and can be particularly valuable for high order systems. Simple numerical rules make this technique suitable for use with a digital computer. The idea behind our method will now be stated briefly. Let k_p, k_i and k_d be proportional, integral, and derivative gains of the PID controller respectively, and $p_{1,2}$ the desired dominant pair of the closed loop poles. Since the chosen poles satisfy the characteristic equation of the closed loop system, it is possible to express integral and derivative terms k_i and k_d as linear functions of the proportional gain k_p, i.e. $k_i = k_i(k_p)$ and $k_d = k_d(k_p)$. This gives rise to k_p as the only free parameter and D-decomposition procedure can now be used to find values of k_p which guarantee the dominance of the chosen poles $p_{1,2}$. In general, the D-decomposition technique is used to determine stabilizing regions in the controller parameter space by a graphical approach. In this paper, the stabilizing region in k_p parameter space which ensures the dominance of the assigned poles is obtained. A very fast and efficient way of calculating the controller parameters is provided.

As stated above, the aim of this work is to improve the system control performances by applying fractional order PID controller. To accomplish this, a procedure based on the D-decomposition technique is used. The primary design goal is to obtain good load disturbance response by minimizing the integrated absolute error. In order to provide the required level of robustness, constraints on the maximum sensitivity M_S and sensitivity to noise measurement M_N are imposed. Also, a good set point response is one of the requirements of closed loop design. To the best of the authors' knowledge, there is no method available in the literature for dominant pole placement with fractional order PID controllers using the D-decomposition method.

The paper is organized as follows. First, a short introduction to fractional calculus and fractional order control is given in Section 2. Statement of the problem and preliminaries are also presented in this Section. In Section 3 D-decomposition method is utilized for solving the problem of dominant pole placement with fractional order PID controllers. Comparisons between classical and fractional order controllers are presented. Numerical simulations are given in Section 4 to illustrate the effectiveness of this method. Section 5 concludes the paper.

2. Essentials of the fractional calculus and statement of the problem

The fractional calculus is theory of integrals and derivatives of arbitrary order, i.e. orders other than integer [21,22].It unifies and generalizes the notions of integer order differentiation and integration. In the last few decades the field of fractional calculus has attracted interest of researchers in several areas including mathematics, physics, engineering etc. [23,24]. There are many proposed definitions of fractional derivatives and integrals. The Riemann-Liouville fractional differ-integral, as the most common definition in the literature, is defined by:

$$ {}_aD_t^p f(t) = \frac{1}{\Gamma(n-p)}\frac{d^n}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{p-n+1}}d\tau, \tag{1}$$

where$(n-1 < p < n)$ and $\Gamma(\bullet)$ is Euler's gamma function. Laplace transform is commonly used to describe the fractional integro-differential operation in the complex domain [25]. It opens the door for the frequency domain analysis. Depending on the adopted definition, the Laplace transform of the fractional derivative varies. For example, the Laplace transform of RL fractional differ-integral ${}_0D_t^p f(t)$ is:

$$ L\left[{}_0D_t^p f(t)\right] = s^p F(s), \quad p \le 0, \tag{2}$$

$$ L\left[{}_0D_t^p f(t)\right] = s^p F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{p-k-1} f(0), \quad p > 0. \tag{3}$$

Fractional control is one of the areas that has been the object of extensive publishing over the recent years. In theory, control systems can include both the fractional order plant model and fractional order controller. The idea of using fractional order algorithms for the control of dynamical systems belongs to A. Oustaloup. He developed the so called CRONE controller [26] and demonstrated its advantage over the PID controller. Here, fractional order controller is introduced to enhance control system performances. Podlubny in his work [6] proposed a generalization of the PID controller, which is called $PI^\beta D^\alpha$ controller. It involves an integrator of orderβ, and differentiator of orderα, wherein α and β are positive real numbers. By taking $\alpha = \beta = 1$, the classical PID