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Research article

Guaranteed cost consensus protocol design for linear multi-agent systems with sampled-data information: An input delay approach

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ABSTRACT

To investigate the energy consumption involved in a sampled-data consensus process, the problem of guaranteed cost consensus for sampled-data linear multi-agent systems is considered. By using an input delay approach, an equivalent system is constructed to convert the guaranteed cost consensus problem to a guaranteed cost stabilization problem. A sufficient condition for guaranteed cost consensus is given in terms of linear matrix inequalities (LMIs), based on a refined time-dependent Lyapunov functional analysis. Reduced-order protocol design methodologies are proposed, with further discussions on determining sub-optimal protocol gain and enlarging allowable sampling interval bound made as a complement. Simulation results illustrate the effectiveness of the theoretical results.

1. Introduction

In the past decade, the cooperative control of multi-agent systems has attracted compelling attention due to its widespread applications in spacecraft, vehicle formations, robotic teams and sensor networks [1–4], etc. The consensus problem, which requires all the agents reach an agreement on certain interest by using local relative information only, has been defined and formulated as a cooperative control problem in [5,6].

Most early studies regarding the consensus problem address the simple case where the agents are often restricted to be single, double or high-order integrators, with extensive consensus conditions established in [3,7,8]. To consider a more general case, consensus of multi-agent systems with linear time invariant (LTI) node dynamics were investigated in [9,10], where the notion consensus region was introduced into protocol design. The results were then extended to synchronization of Lipschitz-type nonlinear dynamical networks in [11]. Instead of focusing on more complex node dynamics, remarkable efforts have been made to improve the performance of a consensus process. For instance, to achieve finite-time consensus in the absence or in the presence of external disturbances, carefully designed consensus algorithms were proposed in [12,13] based on finite-time backstepping control technique. For more details about consensus design of continuous-time multi-agent systems, the survey paper [14] and references therein are recommended.

Note that most of the aforementioned literature on continuous-time multi-agent systems assume that the relative information is available for neighboring agents continuously, since the designed controllers are in a continuous-time structure. However, persistent interactions or remote sensing may be interrupted due to the unreliability of communication channels and the limitations of sensing abilities of agents. Hence, it is more practical to consider intermittent information transmission. Based on the above observation, sampled-data setting was introduced in [15–17], for second-order multi-agent systems under both fixed and time-varying topologies. By using zero-order holders for discretization, sufficient consensus conditions for sampled-data multi-agent systems were given in [11,18–21].

Though the communication and sensing loads can be reduced under the sampled-data setting, the control input energy and the state constraints are not considered. It motivates us to consider sampled-data implementation and guaranteed cost simultaneously for the consensus process. Fortunately, the guaranteed cost consensus problem for multi-agent systems with continuous-time information setting has been formulated and investigated recently, by taking a consensus LQR cost function. Sufficient guaranteed cost consensus conditions are given in [22–24]. Moreover, guaranteed cost consensus seeking for multi-sensor

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networks with Markov switching topologies was investigated in [25], with the motivation to reduce the energy consumption during consensus process. These meaningful works help us a lot in formulating the guaranteed cost consensus problem for multi-agent systems with sampled-data information.

It should be noted that the majority of existing literature on sampled-data multi-agent consensus and guaranteed cost consensus focus on the case where the agents are governed by integrator-type dynamics. In this study, we consider the guaranteed cost consensus protocol design problem for sampled-data multi-agent systems with general linear node dynamics and directed graphs. To the best of the authors' knowledge, there are few research results considering sampled-data implementation and guaranteed cost consensus simultaneously. To solve this problem, a reduced-order equivalent system is constructed to convert the guaranteed cost consensus problem to a guaranteed cost stabilization problem. A refined time-dependent Lyapunov functional [26] is then adopted to conduct the guaranteed cost analysis for the equivalent system. Moreover, guaranteed cost consensus condition and protocol design methods are established in terms of LMIs with the computational burden taken into account. The main contributions of the paper can be summarized as follows: (i) the concept of guaranteed cost is introduced into sampled-data multi-agent systems by considering general linear node dynamics and directed graphs. (ii) A reduced-order equivalent system is constructed to convert the guaranteed cost consensus problem to the guaranteed cost stabilization problem. (iii) A refined sampled-data guaranteed cost consensus condition is given, which can be used to enlarge the allowable sampling interval bound for the sampling instants. (iv) A system decomposition approach is proposed for undirected graph case, so that the protocol can be synthesized via a numerically efficient LMI design.

The rest of the paper is organized as follows. In Section 2, we give some preliminaries on graph theory and LMI facts. In Section 3, we formulate the guaranteed cost consensus problem for sampled-data multi-agent systems. We then present our main results concerning guaranteed cost consensus condition and protocol design methodology in Sections 4 and 5. Numerical studies are carried out in Section 6 to validate our design and a brief concluding remark is drawn in Section 7.

Notations: Throughout this paper, $\mathbb{R}$ denotes the field of real numbers. $\mathbb{R}_+$ and $\mathbb{N}$ denote the set of positive real numbers and nonnegative integers, respectively. $\mathbb{R}^{n\times m}$ denotes the space of real $n\times m$ matrices. I_N and 0_N represent, respectively, the identity matrix and the zero matrix of dimension N. $\mathbf{1}_N$ and $\mathbf{0}_N$ are defined as $\mathbf{1}_N=[1,\ldots,1]^T\in\mathbb{R}^N$ and $\mathbf{0}_N=[0,\ldots,0]^T\in\mathbb{R}^N$. $\|x\|$ denotes the 2-norm of a vector x. $\mathrm{diag}\{A_1,\ldots,A_n\}$ represents a block-diagonal matrix with square matrices A_i, $i=1,\ldots,n$, on its diagonal. $\mathrm{col}\{a_1,a_2,\ldots,a_N\}$ denotes the column vector in the form of $\left[a_1^T,a_2^T,\ldots,a_N^T\right]^T$. The matrix inequality $A>B$ means that A and B are square Hermitian matrices, and $A-B$ is positive definite. $A\otimes B$ represents the Kronecker product of matrices A and B. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Preliminaries

Let $\mathcal{G}=(\mathcal{V},\mathcal{E})$ be a directed graph, where $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$ is a nonempty, finite set of nodes, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges represented by ordered pairs of distinct nodes. A directed edge in $\mathcal{G}$ with parent node v_i and child node v_j is represented by $(v_i,v_j)\in\mathcal{E}$. Besides, a directed path of $\mathcal{G}$ from node v_{s_1} to node v_{s_l} is a sequence of edges of the form $(v_{s_k},v_{s_{k+1}})$, $s_k\in 1,\ldots N$, $k=1,\ldots,l-1$. Undirected graphs are defined as special cases of directed graphs such that $(v_i,v_j)\in\mathcal{E}$ implies $(v_j,v_i)\in\mathcal{E}$, and an undirected path in $\mathcal{G}$ is defined analogously to a directed path.

The graph adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined such that $a_{ii}=0$, $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Since a_{ij} denotes the weight for the edge (v_j,v_i), it can be assigned as 1 if $(v_j,v_i)\in\mathcal{E}$ and the corresponding weight are not relevant. If $\mathcal{G}$ is undirected, a_{ij} is assumed to equal to a_{ji} for $\forall\, i\neq j$. The Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ is defined as $l_{ii}=\sum_{j\neq i}a_{ij}$ and $l_{ij}=-a_{ij}$, $i\neq j$.

Definition 1 (*Directed spanning tree*). A directed graph $\mathcal{G}$ is said to have a directed spanning tree, if and only if $\mathcal{G}$ has at least one node with a directed path to all other nodes.

Remark 1. For an undirected graph $\mathcal{G}$, the existence of a directed spanning tree is equivalent to being connected. Thus the notion connected is used for the undirected graph $\mathcal{G}$.

Lemma 1 ([27]). *Let $\mathcal{L}\in\mathbb{R}^{N\times N}$ be the Laplacian matrix of a directed graph $\mathcal{G}$, then the following statements hold: (i) $\mathcal{L}$ has at least one zero eigenvalue with $\mathbf{1}_N$ as the corresponding right eigenvector and all nonzero eigenvalues have positive real parts, (ii) zero is a simple eigenvalue of $\mathcal{L}$, and there exists a unique nonnegative left eigenvector r of $\mathcal{L}$ associated with the zero eigenvalue, satisfying $r^T\mathcal{L}=0$ and $r^T\mathbf{1}_N=1$, if and only if $\mathcal{G}$ has a directed spanning tree.*

Proposition 1 (*Jensen's Inequality [26]*). *For any $n\times n$ matrix $R>0$, scalar $h>0$ and a vector function ϕ: $[-h,0]\to\mathbb{R}^n$ such that the integrations concerned are well-defined, the following holds*:

$$\int_{-h}^{0}\phi^T(s)R\phi(s)ds\geq\frac{1}{h}\int_{-h}^{0}\phi^T(s)dsR\int_{-h}^{0}\phi(s)ds. \tag{1}$$

3. Problem formulation

We start by considering a multi-agent system consists of N agents in the form

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad i\in\mathcal{I}, \tag{2}$$

where $x_i\in\mathbb{R}^n$, $u_i\in\mathbb{R}^m$ are the state, input of agent i, and $\mathcal{I}=\{1,\ldots,N\}$. A and B are constant matrices with compatible sizes, and the pair (A,B) is assumed to be stabilizable.

The information exchanges among these agents are modeled by digraphs. Let ν_i denote the i-th agent, and the communication graph among all the N agents is defined such that $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{\nu_1,\nu_2,\ldots,\nu_N\}$, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. A directed edge $(\nu_i,\nu_j)\in\mathcal{E}$ implies that ν_j can obtain information from ν_i, and ν_i is a neighbor of ν_j. Self-loops in the form of (ν_i,ν_i) are excluded from $\mathcal{E}$ when modeling. Let $\mathcal{N}_i$ denote the neighborhood index set of the i-th agent with $\mathcal{N}_i=\{j,(\nu_j,\nu_i)\in\mathcal{E},i\neq j\}$.

In consensus design of linear multi-agent systems, linear relative-state feedback consensus law is widely used, due to its fundamentality and effectiveness. A general form of consensus laws of this type is considered in this paper, which is given as follows:

$$u_i(t)=K\sum_{j\in N_i}\left(x_i(t)-x_j(t)\right). \tag{3}$$

For the sake of simplicity, let e_i denote the neighborhood synchronization error with respect to the i-th agent, that is, $e_i(t)=\sum_{j\in N_i}(x_i(t)-x_j(t))$, then (3) can be written as,

$$u_i(t)=Ke_i(t). \tag{4}$$

To ensure that the above scheme solves the consensus problem for multi-agent systems, persistently remote sensing or interactions

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