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Research article

Performance limitations of networked control systems with quantization and packet dropouts ☆

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ABSTRACT

This paper investigates the problem of optimal tracking performance of networked control systems (NCSs) with quantization and packet-dropouts. The system under consideration is linear time-invariant (LTI), multi-input multi-output (MIMO), where an H_2 norm of error signal between the reference input and the system output is used as the tracking performance index. The impacts of packet-dropouts in the communication channel and the quantized input and output are studied. The goal is to obtain the minimal error in tracking a random signal, by searching through all possible stabilizing two-parameter controllers. It is shown that, the minimum value of tracking error is closely related to the reference input signal direction, the non-minimum phase zeros and unstable poles of the given plant, including the locations and directions. We also demonstrated the quantization error and the packet-dropouts may degrade the tracking performance. A typical example is given to evaluate the theoretical results.

1. Introduction

In the past decade, the performance limitations of networked control systems (NCSs) has gained considerable attention [1,2], which can help researchers to understand how the performance of the systems may be intrinsically constrained by the characteristics of the plant. For tracking performance of the classical systems, it is generally known that the minimal tracking error depends upon the non-minimum phase zero, the unstable poles and the time delays in the plant [3].

NCSs with distributed sensors, controllers and actuators have been produced as the rapid development of network technology [4–6]. These systems have significant advantages in engineering applications such as networked direct current motor and tele-medicine [7,8]. However, the controllers and the plants to be controlled in the NCSs often communicate with each other in a non-ideal manner due to long distance communication channels. Thus many factors such as time-delay [9,10], packet-dropouts [11–13], quantization [14,15] will inevitably bring some adverse effects on the performance of system, and even worse they may cause the systems instability. In [16], assuming only one node can access the network and send its information the authors investigate the stability of NCSs with time-varying transmission intervals and time-varying transmission delays. A new linear delayed delta operator switched system model has been proposed in [17] to describe the networked control systems with packet-dropouts and network-induced delays, and a verification theorem has been given to show the solvability of the stabilization conditions by solving a class of finite linear matrix inequalities (LMIs). Other related works can also be found in [18–20].

The results obtained in previous works have provided valuable insight into about the relationship between stability, performance and communication constraints. However, it should be noted that signal quantization is an essential part of the communication process, and packet-dropouts are typical features associated with the networked control systems. Thus, it is meaningful and significant to reveal the quantitative relationship between the minimal tracking error and communication constraint. The goal of this work is to adopt the two-parameter controllers to investigate the optimal tracking performance of the networked control systems with quantization and packet-dropouts. The tracking performance can be measured by the energy of the error signal between the output of the plant and the reference signal. Two cases are considered in this paper. In the first case, the system sensor is far away from the plant while the controller is near to the plant. In the second case, the setting is the opposite. The adopted model can be

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found in many real systems. For example, in a robot surgery system with remote monitoring, the patient is the plant and the robot is the controller. The remote expert obtains information via the network transmission, and the instruction of the expert is then returned to the robot via the network transmission. Main contributions of this paper can be summarized as follows. Firstly, explicit expressions are given to show the relationship between tracking performance and intrinsic characteristics of the plant. Secondly, by using additive white noise to model the quantization error, and a binary stochastic process to model the packet the quantization and packet dropouts' effects on tracking performance are quantitatively revealed. Thirdly, the results obtained in this paper may give some guidance for the design of optimal controllers.

The remainder of this paper is organized as follows. In Section 2, the notations are defined, the Youla parameterization of stabilizing controllers is introduced, and a brief narrative of all-pass factors of the non-minimum phase transfer function matrices is provided. Section 3 provides the formulation and solution of the problem of optimal tracking performance with quantization output and packet-dropouts. In Section 4, the optimal tracking performance with quantization input and packet-dropouts is further investigated. An illustrative example is given in Section 5. Finally, the paper is concluded in Section 6.

2. Preliminaries

The notations used throughout in this paper are described as follows. $\bar{z}$ denotes the conjugate of a complex number z. The transpose and conjugate transpose of a vector u and a matrix A are denoted by u^T, u^H and A^T, A^H, respectively. The open unit disc is denoted by $D:=\{z\in C: |z|<1\}$, the closed unit disc is denoted by $\bar{D}:=\{z\in C: |z|\le 1\}$, the unit circle is denoted by $\partial D:=\{z\in C: |z|=1\}$, and the complement of $\bar{D}$ by $\bar{D}^c:=\{z\in C: |z|>1\}$. Moreover, let $\|\cdot\|_2$ denote the Euclidean vector norm and $\|\cdot\|_F$ the Frobenius norm, $\|G\|_F^2:=\mathrm{tr}(G^HG)$. The Hilbert space $\mathcal{L}_2$ is defined as

$$\mathcal{L}_2:=\left\{G:\ G(z)\ \text{measurable in}\ \partial D,\ \|G\|_2:=\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\|G(e^{j\theta})\|_F^2\,\mathrm{d}\theta\right)^{1/2}<\infty\right\},$$

with the inner product $\langle F, G\rangle:=\frac{1}{2\pi}\int_{-\pi}^{\pi} tr(F^H(e^{j\theta})G(e^{j\theta}))\mathrm{d}\theta$, where F and G are the transfer function matrices in the Hilbert space. It is well known that $\mathcal{L}_2$ admits an orthogonal decomposition into the subspaces $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$, where

$$\mathcal{H}_2:=\left\{G:\ G(z)\ \text{analytic in}\ \bar{\mathrm{D}}^c,\ \|G\|_2:=\sup_{r>1}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\|G(re^{j\theta})\|_F^2\,\mathrm{d}\theta\right)^{1/2}<\infty\right\},$$

and

$$\mathcal{H}_2^{\perp}:=\left\{G:\ G(z)\ \text{analytic in D},\ \|G\|_2:=\sup_{r<1}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|G(re^{j\theta})\right\|_F^2\,\mathrm{d}\theta\right)^{1/2}<\infty\right\}.$$

It follows that for any $F\in\mathcal{H}_2$ and $G\in\mathcal{H}_2^{\perp}$, we have $\langle F, G\rangle=0$. It is worth pointing out that the same notation $\|\cdot\|_2$ will be used to denote these norms, and the meaning of each of these norms will be cleared from the context. Let $\mathbb{R}\mathcal{H}_\infty$ denote the set of all stable, proper, and rational transfer function matrices. The expectation operator is denoted by $E\{\cdot\}$. Finally,

$$\cos\angle(u, v):=\frac{|u^Hv|}{\|u\|\|v\|}.$$

where $\angle(u, v)$ is the principal angle between the two subspaces spanned by u and v.

In this section, some important factorizations are described that will be frequently used. The packet-dropouts' probability in the communication channel is denoted by α and thus $0\le\alpha<1$. For the rational transfer function matrix $(1-\alpha)G(z)$, the right and the left coprime factorizations are given by

$$(1-\alpha)G(z)=NM^{-1}=\tilde{M}^{-1}\tilde{N}, \tag{2.1}$$

where $N, M, \tilde{N}, \tilde{M}\in\mathbb{R}\mathcal{H}_\infty$ and satisfy the double Bezout identity

$$\begin{bmatrix}\tilde{X} & -\tilde{Y}\\ -\tilde{N} & \tilde{M}\end{bmatrix}\begin{bmatrix}M & Y\\ N & X\end{bmatrix}=I, \tag{2.2}$$

for some $X, Y, \tilde{X}, \tilde{Y}\in\mathbb{R}\mathcal{H}_\infty$. Then, all the stabilizing two-parameter compensators K can be characterized by the following set [21]:

$$\mathcal{K}:=\left\{K:\ K=[K_1\ \ K_2]=\left(\tilde{X}-R\tilde{N}\right)^{-1}\left[\Theta\ \ \tilde{Y}-R\tilde{M}\right],\ \Theta, R\in\mathbb{R}\mathcal{H}_\infty\right\}. \tag{2.3}$$

where K_1 and K_2 are two independent controllers that will be designed. Assume that G is right invertible, which implies that $G(z)$ has a right inverse for some z. For a right-invertible $G(z)$, each of its nonminimum phase zeros is also one for $N(z)$. Denote $s_i\in\mathbb{C}_+$, $i=1,\ldots,N_s$ as the non-minimum phase zeros of $G(z)$, where N_s is the number of non-minimum phase zeros, and η_i are the corresponding unitary zero direction vectors. Then it is possible to factorize $N(z)$ as

$$N(z)=L(z)N_m(z), \tag{2.4}$$

where $L(z)$ is an all-pass factor and $N_m(z)$ is the minimum phase part of $N(z)$. A useful all-pass factor is given by

$$L(z)=\prod_{i=1}^{N_s}L_i(z),\ L_i(z)=\left[\eta_i\ \ U_i\right]\begin{bmatrix}\dfrac{1-\bar{s}_i}{1-s_i}\dfrac{z-s_i}{1-\bar{s}_iz} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\eta_i{}^H\\ U_i^H\end{bmatrix}, \tag{2.5}$$

where U_i are matrices that together with η_i form a unitary matrix. If the plant $G(z)$ has unstable poles $p_k\in\mathbb{C}_+$, $k=1,\ldots,N_p$, where N_p is the number of the unstable poles and Λ is a real diagonal matrix, then it is possible to factorize $\tilde{M}(z)\Lambda$ as

$$\tilde{M}(z)\Lambda=\tilde{M}_m(z)\tilde{B}(z), \tag{2.6}$$

where $\tilde{M}_m(z)$ is the minimum phase and $\tilde{B}(z)$ is an all-pass factor. Specifically, $\tilde{B}(z)$ can be constructed as follows:

$$\tilde{B}(z)=\prod_{k=1}^{N_p}\tilde{B}_k(z),\quad \tilde{B}_k(z)=\left[w_k\ \ W_k\right]\begin{bmatrix}\frac{z-p_k}{1-\bar{p}_kz} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}w_k^H\\ W_k^H\end{bmatrix},$$

where w_i are unitary vectors obtained by factorizing the zeros one at a time, and W_i are the matrices that together with w_i form a unitary matrix. The tracking performance index of the system is defined as

$$J:=E\left\{e(k)^Te(k)\right\}, \tag{2.7}$$

where $e(k)=y(k)-r(k)$. The minimum tracking error achievable by all possible stabilizing controllers is then determined as

$$J^*:=\inf_{K_1,K_2\in\mathcal{K}}J,$$

where $\mathcal{K}$ denotes the set of all stabilizing two-parameter controllers.

3. Tracking performance with quantization output and packet dropouts

In the section, the problem under consideration is depicted in Fig. 1, in which G denotes the plant and $[K_1\ K_2]$ are the two-parameter controllers. Q is used to model the uniform quantizer, which takes uniform quantization interval as shown in Fig. 2. The quantizer is a crucial part in the process of signal transmission. d_r