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Research article

Semi-global output consensus of discrete-time multi-agent systems with input saturation and external disturbances

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ABSTRACT

This paper investigates the problem of leader-following output consensus of a linear discrete-time multi-agent system with input saturation and external disturbances. Low-gain state feedback technique and output regulation theory are used to deal with the output consensus of multi-agent systems with input saturation and external disturbances. Both the cases with identical and non-identical disturbances are discussed in the multi-agent systems. For the case of identical external disturbance, the output consensus can be attained when the directed graph has no loop and there exists at least one directed path from the leader to every follower agent. For the case of non-identical external disturbances, the output consensus can be achieved if the directed graph is strongly connected and detailed balanced, and at least one follower can have access to the information of the leader. Numerical simulation results are presented to demonstrate the validation of the proposed design.

1. Introduction

In recent years, cooperative control of multi-agent systems has already attracted quite a bit of attention due to its widely application in bioinformatics, sensor networks, unmanned aircraft vehicles, and so on [1–3]. According to the control requirements, there are many different cooperative control protocols such as consensus, synchronization, formation control, and flocking [4–8]. In particular, consensus of multi-agent systems has been one of the most concerned problems of cooperative control. The consensus problem is to make the state of each agent in a system agree on the same value via appropriate distributed control protocols [9], which has been investigated in different agent dynamics. The multi-agent system with single-integrator dynamics was studied at first [10–12]. Thereafter, some researchers extended the work on the single-integrator dynamics to the cases with linear dynamics [13–15], and with nonlinear dynamics [16].

It is well known that input saturation is inevitable in physical systems, for example, the pump is limited by the maximum working capacity, the horizontal steering of the aircraft is limited by the vertical stabilizer, and so on [17]. Therefore, the actuator saturation should be taken into account in the multi-agent systems. The issues of leader-following linear multi-agent systems with input saturation have been investigated in recent years [18–21]. Among these efforts, the low-gain state feedback and output feedback control protocols have been proposed to attain the leader-following consensus of multi-agent systems with input saturation via restricting the parameters of the algebraic Riccati equation(ARE), respectively [18,19]. After that, the distributed control protocols and the Lyapunov function method have been applied to investigate the leader-following consensus problem of discrete-time systems with saturation constraints by adjusting the parameters of the modified algebraic Riccati equation(MARE) [20,21]. Different from the results that the full state of the leader can be tracked by the followers, the output consensus of continuous-time multi-agent systems with input saturation and external disturbances have been studied for both undirected and directed networks, respectively [22,23].

In this paper, we study the output consensus of the discrete-time multi-agent systems in the presence of input saturation and external disturbances. Both the cases with the identical and non-identical external disturbances are discussed, respectively. The contributions of this paper are as follows. First, we extend the existing highly related results to a more practical situation, i.e., the multi-agent systems in the presence of both input saturation and external disturbances. Second, we extend the work about the output consensus of the continuous-time multi-agent systems subject to input saturation and external disturbances in [23] to the discrete-time multi-agent systems. Moreover, the algorithm design and the proof method of stability in this paper are completely

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different from the continuous-time case.

This paper is composed of the following parts. In Section 2, we give an introduction to the basics of the graph theory and the linear dynamics of the discrete-time multi-agent systems, and propose the problem that we need to deal with. Next, we discuss the discrete-time systems with the identical external disturbance and non-identical external disturbances in Section 3 and 4, respectively. In Section 5, Numerical simulation results are presented to demonstrate the validation of the proposed protocols. A brief conclusion is given in the end.

The following notations will be used in this paper. $\|x\|$, and $sup(f(\bullet))$ denote the Euclidean norm, or 2-norm, of $x \in R^n$ and the supremum of the function $f(\bullet)$, respectively. $col\{\ \}$ denotes the set of the column vectors. $sup_{k\geq K_1}\|x(k)\|_\infty$ denotes the supremum of the Euclidean norm of $x(k)$, for any $K_1 \geq 0$ and any $x(k) \in R^n$. $L\nu(c)$ is the invariant set [24].

2. Preliminaries and problem statement

Consider a group of N agents with general linear dynamics. The motion of each agent, labeled from 1 to N, is given as follows:

$$\begin{cases} x_i(k+1) = Ax_i(k) + B\sigma(u_i(k)) + E_i\omega(k), \\ y_i(k) = Cx_i(k), \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(k) \in \mathbf{R}^n$, $u_i(k) \in \mathbf{R}^m$ and $y_i(k) \in \mathbf{R}^q$ are the state, the control input and the output of agent i, respectively; A, B, C are constant matrices with appropriate dimensions; $E_i\omega(k) \in \mathbf{R}^n$ is the external disturbances described in detail later, and $\sigma(u_i(k))$ is a saturation function defined as:

$$\sigma(u_i(k)) = [\sigma(u_{i1}(k)),\ \sigma(u_{i2}(k))\ldots,\ \sigma(u_{in}(k))]^T,$$
$$\sigma(u_{ij}(k)) = \mathrm{sign}(u_{ij}(k))\min\{|u_{ij}(k)|,\ \varpi\}$$

where ϖ is a positive constant, and,

$$\mathrm{sign}(u_{ij}(k)) = \begin{cases} 1, & \text{if } u_{ij}(k) > 0 \\ -1, & \text{if } u_{ij}(k) \leq 0. \end{cases}$$

It is worth noting that the dynamics of the agents in (1) are not identical in general, because E_i, $i = 1, 2, \ldots, N$ may be different. We will define this case as the non-identical disturbance case because maybe the disturbances enter each agent i differently. Similarly, for the case that the dynamics of the agents are identical, that is, $E_1 = E_2 = \cdots = E_N$, we define it as the identical disturbance case.

The kinetics of the leader, labeled as v_0, is given as follows:

$$\begin{cases} \omega(k+1) = S\omega(k), \\ y_0(k) = -Q\omega(k), \end{cases} \tag{2}$$

where $\omega(k) \in \mathbf{R}^l$ is the state, $y_0(k) \in \mathbf{R}^q$ is the output of the leader, and S, Q are constant matrices with appropriate dimensions. Moreover, the leader generates both a reference signal $y_0(k)$ to be tracked and the disturbance signal $E_i\omega(k)$ to be rejected by the followers.

The problem of semi-global output consensus of discrete-time multi-agent systems given in (1) and (2) is described as follows: for any given bounded sets $X_0 \in \mathbf{R}^n$ and $W_0 \in \mathbf{R}^l$, if $\omega_0 \in W_0$ and $x_i(0) \in X_0$, $i = 1, 2, \ldots, N$, design a linear feedback control protocol $u_i(\cdot)$ for each agent i by only utilizing local information from its neighbors, such that the multi-agent systems can reach leader-following output consensus, that is,

$$\lim_{k\longrightarrow\infty} \|e_i(k)\| = 0, \quad i = 1, 2, \ldots, N, \tag{3}$$

where $e_i(k) = y_i(k) - y_0(k)$.

Assumption 1 *[17]*. Suppose that the pair (A,B) is asymptotically null controllable with bounded controls(ANCBC), that is,

(1) (A,B) is stabilizable;.
(2) All eigenvalues of A are inside or on the unit circle.

In this paper, the communication topology of system (1) can be described by a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$, where the node set $\mathbf{V} = \{1, 2, \ldots, N\}$ represents the followers of the systems and the edge set $\mathbf{E} = \{(i, j) \in \mathbf{V} \times \mathbf{V}: i \sim j\}$ denotes the neighboring relation among the followers. If agent i has access to the information of agent j, then $(j, i) \in \mathbf{E}$ and agent j is said to be a father of agent i. Denote a directed path between i_1 and i_{q+1} as a series of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_q, i_{q+1})$. If $i_1 = i_{q+1}$, then the digraph $\mathbf{G}$ contains a loop. The set of the father agents of agent i is defined as $\mathcal{N}_i = \{j: (j, i) \in \mathbf{E}\}$. Let $|\mathcal{N}_i|$ be the number of agents in the set $\mathcal{N}_i$. We define the length of the longest directed path from the leader to agent i as f_{0i}. For an integer r, denote the set of followers whose length of the longest directed path from the leader to those is r as $F_r = \{i: f_{0i} = r\}$. Let $|F_r|$ be the amount of the agents in the set F_r.

The adjacency matrix of the graph $\mathbf{G}$ is $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$, where $a_{ij} > 0$ if $(j, i) \in \mathbf{E}$ otherwise $a_{ij} = 0$, moreover, $a_{ii} = 0$ for $i = 1, 2, \ldots, N$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{N\times N}$ of the directed graph $\mathbf{G}$ is defined by $l_{ij} = -a_{ij}$ if $i \neq j$ otherwise $l_{ii} = \sum_{i\neq j} a_{ij}$. If agent i has access to the information of the leader v_0, $a_{i0} = 1$, otherwise $a_{i0} = 0$. Denote $\mathcal{H} = diag\{a_{10}, a_{20}, \ldots, a_{N0}\}$.

Lemma 1 *[20]. Let Assumption 1 hold, $\gamma \in (0, 1]$. Then for each $\varepsilon \in (0, 1]$, there exists an unique positive defined matrix $P(\varepsilon)$ to solve the modified algebraic Riccati equation (MARE)*

$$P = A^TPA - \gamma A^TPB(B^TPB + I)^{-1}B^TPA + \varepsilon I, \tag{4}$$

moreover, $\lim_{\varepsilon\to 0}P(\varepsilon) = 0$.

3. The case of identical external disturbance

In this section, we investigate the leader-following output consensus with the case of identical disturbance.

Let $E_1 = E_2 = \cdots = E_N = E$. Then, the kinetics of the followers also can be written as,

$$\begin{cases} x_i(k+1) = Ax_i(k) + B\sigma(u_i(k)) + E\omega(k), \\ y_i(k) = Cx_i(k), \quad i = 1, 2, \ldots, N. \end{cases} \tag{5}$$

Assumption 2 *[25]*. The directed graph $\mathbf{G}$ contains no loop and the leader has at least one directed path to each follower agent i, $i = 1, 2, \ldots, N$.

Assumption 3 *[17]*. There exist matrices Π, $\Gamma$and H such that.

(a) They are the solutions of

$$\begin{cases} \Pi S = A\Pi + B\Gamma + E, \\ C\Pi + Q = 0, \\ H\Pi - \Gamma = 0. \end{cases} \tag{6}$$

(b) There exist a positive constant $\varrho < \varpi$ and an integer $K_1 \geq 0$, for all $\omega(k)$ subject to $\omega(0) \in W_0$, such that $\|\Gamma\omega(k)\|_{\infty,K_1} \leq \varrho$, where $\|\Gamma\omega(k)\|_{\infty,K_1} := sup_{k\geq K_1}\|\Gamma\omega(k)\|_\infty$.

Lemma 2. *For any two vectors $x, y \in \mathbf{R}^n$, and matrix $\mathcal{M} = (m_{ij}) \in \mathbf{R}^{n\times n}$,*

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