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Model-free adaptive fractional order control of stable linear time-varying systems

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ABSTRACT

This paper presents a new model-free adaptive fractional order control approach for linear time-varying systems. An online algorithm is proposed to determine some frequency characteristics using a selective filtering and to design a fractional PID controller based on the numerical optimization of the frequency-domain criterion. When the system parameters are time-varying, the controller is updated to keep the same desired performances. The main advantage of the proposed approach is that the controller design depends only on the measured input and output signals of the process. The effectiveness of the proposed method is assessed through a numerical example.

1. Introduction

Motivated by the ability to describe the complex behavior of some physical phenomena, fractional calculus has been used in many engineering fields such as modeling of physical systems [1–4], control [5–7] and diagnosis [8].

In control theory, the use of fractional calculus has gained much success among researchers. This great interest is motivated by the robustness properties of the fractional differentiation. Many design methods have been proposed in literature to deal with fractional order control of systems. They are based on time-domain [9–13] or frequency-domain approaches [14–17]. The most known ones are the *Oustaloup*'s CRONE approach [18] and the optimization-based methods initially developed by Monje et al. [15].

Among the developed frameworks, most of the proposed methods in literature are model-based and even if they achieve a good robustness towards gain uncertainties, they are not robust towards the pole uncertainties. On the other hand, these methods require the knowledge of the process model (differential equation, transfer function, state space). However, in many situations, the system model may be unknown and/or time-varying. So, a fixed controller cannot provide an acceptable closed-loop performances in all situations. Thus, it is necessary to retune the controller parameters. In these cases, the classical adaptive control can be used.

Several studies are focused on the application of the adaptive control in the fractional case. The fractional order model reference control (FO-MRAC) and the fractional adaptive control are developed in [19–22,12,13]. But, most adaptive control techniques are typically supposed that the the system structure is a *a priori* known and the parameters are needed to identification step. So, it is necessary to go through the following steps of system experiments: estimation of the parameters, computation of control strategies then, implementation [22,21]. But, that the identification-based control approaches are not very well adapted for system control [23] for many reasons namely: the model convergence, the system stability, the relation between the excitation signal and the system response for control performances, etc. Therefore, in these cases, a model-free adaptive control can be used to avoid any explicit system identification procedure. It uses only the input and output measurement data of the system.

The model-free adaptive control was introduced firstly for rational case (see [24] and the reference therein for an overview) However, despite its effectiveness, this approach has not yet well exploited the robustness of the fractional order controller. Recently, in 2009, a model-free adaptive control in the frequency domain was proposed by Ionescu et al. and applied to mechanical ventilation [25]. But, the mentioned works suppose that the system is linear time-invariant and the fractional order controller is designed based on closed-loop specifications given by the reference model. They use some characteristics of the system, of an integrator or a dead-time, etc. However, these informations about the system may not always be available. In addition, the choice of the excitation signal and its frequency to identify the controller presents a problem.This is because the non-obvious relation between the excitation signal and the system response for control performances [25]. In 2012, Villagra et al. has proposed a model-free

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fractional order PID control which is applied to DC motors in flexible joints [26]. Even if this work is applicable to unstable linear time-unvarying (LTI) systems, a systematic procedure that allows firstly to analytically characterize the stable regions of the closed-loop system, and thereafter to assign a set of specifications to a particular controller configuration, is required.

Within this context, a model-free adaptive fractional order PID control for a stable linear time-varying (LTV) systems is proposed in this paper. It is based on a selective filtering to determine some frequency characteristics (the gain and the phase) of the system and, using the numerical optimization of frequency-domain criterion, the fractional order PID controller is designed. If the system parameters are time-varying, the controller is updated to keep the same desired performances. This approach uses only the input and output measurement data of the process.

In the case of unstable system, the estimation method using selective filters still applicable but the design method should be changed because it is a Bode-based design method which assume that the system is stable in open-loop. This method improves the robustness and ensures the iso-damping property of LTV systems.

The rest of this paper is outlined as follows: Section 2 presents a brief mathematical background and illustrates the problem statement. The fractional order PID controller is presented in Section 3. The model-free adaptive fractional order control design is developed in Section 4. In Section 5, the performances of the proposed scheme are assessed through a numerical example. Finally, conclusions and some perspectives are established in Section 6.

2. Preliminaries

2.1. Fractional calculus

Fractional calculus is a generalization of integer differentiation/integration to a fractional order. There are several definitions of fractional derivative in time-domain. The commonly one used is the Grünwald-Letnikov definition [27].

Using the Grünwald-Letnikov definition, the μ-order fractional differentiation of a continuous-time function $f(t)$, $f(t) = 0$ for $t \le 0$, is given by [27]

$$D^{\mu} f(t) \simeq \frac{1}{h^{\mu}} \sum_{k=0}^{K} (-1)^k \binom{\mu}{k} f(t-kh), \quad \forall\, t \in \mathbb{R}_+, \tag{1}$$

where $D = \left(\frac{d}{dt}\right)$ is the time domain differential operator, h is the sampling period, $t = Kh$ and $\binom{\mu}{k}$ is the Newton's binomial generalized to the fractional orders, such as

$$\binom{\mu}{k} = \begin{cases} 1 & \text{if } k = 0, \\ \dfrac{\mu(\mu-1)(\mu-2)\ldots(\mu-k+1)}{k!} & \text{if } k > 0 \end{cases}. \tag{2}$$

Under zero initial conditions, the Laplace transform of μ-order fractional differentiation of a continuous-time function $f(t)$ is given by

$$\mathcal{L}\left\{D^{\mu} f(t)\right\} = s^{\mu} F(s), \tag{3}$$

where $F(s) = \mathcal{L}(f(t))$ and s is the Laplace variable or the differentiator operator.

A SISO fractional order system can be governed by the following fractional differential equation

$$\sum_{n=0}^{N} a_n \mathrm{D}^{\alpha_n} y(t) = \sum_{m=0}^{M} b_m \mathrm{D}^{\beta_m} y_s(t), \tag{4}$$

where $y_s(t)$ and $y(t)$ are respectively the input and the output

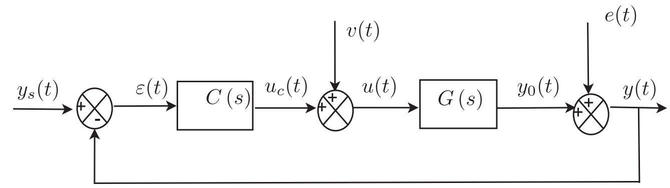

Fig. 1. Fractional closed-loop time-varying system.

signals and $(a_n, b_m) \in \mathbb{R}$ are the linear coefficients of the differential equation. The fractional orders α_n and β_m are allowed to be non-integer positive numbers.

Applying the Laplace transform to the fractional differential Eq. (4) yields to the fractional transfer function defined by

$$G(s) = \frac{B(s)}{A(s)} = \frac{\sum_{m=0}^{M} b_m s^{\beta_m}}{\sum_{n=0}^{N} a_n s^{\alpha_n}}. \tag{5}$$

2.2. Problem statement

Consider the fractional closed-loop of time varying-system depicted in Fig. 1.

$G(s)$ is the process transfer function which is supposed unknown and time-varying. $C(s)$ is the fractional order controller transfer function. $y_s(t)$ and $y_0(t)$ are respectively the set-point input and the continuous-time noise-free output.

The measurable output signal $y(t)$ is eventually corrupted by an additive white noise $e(t)$ such that

$$y(t) = y_0(t) + e(t). \tag{6}$$

The control signal is given by

$$u(t) = u_c(t) + v(t), \tag{7}$$

where $u_c(t)$ is the controller output and $v(t)$ is an additive input disturbance.

Consider the fractional open-loop transfer function defined by

$$L(s) = C(s) G(s), \tag{8}$$

and the fractional closed-loop transfer function given by

$$T(s) = \frac{C(s) G(s)}{1 + C(s) G(s)}. \tag{9}$$

If the system model is known, a closed-loop system identification techniques developed by Yakoub et al. [1] can be used to estimate the system parameters and then design a suitable controller. However, if the model is unknown and/or its parameters are time-varying, a fractional order adaptive control as the Fractional Order-Model Reference Adaptive Control (FO-MRAC), developed in [13,12,25], can be used. But, for complex real systems, the system model is often difficult to estimate. To overcome this problem, a model-free adaptive control without a parametric identification is developed in the next section. It is based on selective filters to determine only some frequency characteristics required for the controller optimization-based design. The main feature of this approach is its implementation simplicity and its effectiveness. The controller design uses only on the measured input and output signals of the process.

In this following, the controller $C(s)$ is a fractional proportional integral derivative (PID) controller.

3. Fractional order PID controller

Recently, *Podlubny* has proposed a fractional order PID

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