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Research article

Predictor-based control for an inverted pendulum subject to networked time delay

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ABSTRACT

The inverted pendulum is considered as a special class of underactuated mechanical systems with two degrees of freedom and a single control input. This mechanical configuration allows to transform the underactuated system into a nonlinear system that is referred to as the normal form, whose control design techniques for stabilization are well known. In the presence of time delays, these control techniques may result in inadequate behavior and may even cause finite escape time in the controlled system. In this paper, a constructive method is presented to design a controller for an inverted pendulum characterized by a time-delayed balance control. First, the partial feedback linearization control for the inverted pendulum is modified and coupled with a state predictor to compensate for the delay. Several coordinate transformations are processed to transform the estimated partial linearized system into an upper-triangular form. Second, nested saturation and backstepping techniques are combined to derive the control law of the transformed system that would complete the design of the whole control input. The effectiveness of the proposed technique is illustrated by numerical simulations.

1. Introduction

Systems having fewer control inputs than degrees of freedom currently available are commonly known as underactuated systems. This type of mechanical systems provides a challenging area for control system design. The challenge basically stems from the fact that underactuated systems fail the Brockett's necessary conditions to be controlled using continuous state feedback design and thus standard methods originally designed for fully actuated mechanical systems such as feedback linearization are unsuccessful. The mechanical system under discussion in this paper is the Cart-Pole system, which is a benchmark that consists of a rod attached to a cart by means of a pivot. Given the ability to the pendulum to rotate freely in the *xy* plane, the control objective of such mechanical system is to bring the pendulum to the upright unstable position by moving the cart in the horizontal plan. Since the angular acceleration is not directly controlled, this makes of the Cart-Pole an underactuated system. Several control strategies for swinging up and stabilization problem have been extensively studied in the past decade (see for instance, [1–7]). The current trend in control engineering is to establish new strategies that allow remote control of different robotic systems; including mobile robots [8,9] and manipulators [10]. These type of systems are usually linked via a delay-induced wireless communication channels, which may compromise the performance and the stability of the controlled system. Motivated by the measurements of the network delays induced by the internet connection, we will consider the case of constant delays for the forward as well as for the backward time delay. Several techniques have been proposed in the context of teleoperation to get around the negative effect of the induced time delay (see [11] for an overview of different proposed methods). All these methods have nonetheless been applied to fully actuated mechanical systems. Few attempts however have been given to analyze the effect of time-delays that arise in the input of an underactuated system [12]. Motivated by this circumstances, this paper mainly focuses on a state predictor scheme that anticipates the delay in the feedback control for the stabilization of the Cart-Pole system.

The Cart-Pole system has received extensive attention by the control community to obtain deep insight into the way the control algorithms can be derived for the stabilization of the cart position and the tilt angle of the pendulum. The control design can be categorized in fact into two classes. The first class includes algorithms making use of the energy shaping approach [1,2,4], etc. While for the second class the pendulum is exposed to swinging oscillations until it reaches the homoclinic orbit, once in it, a linear controller is switched on to ensure the pendulum angle converges

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to zero [13,7]. None of the aforementioned approaches however have considered the effect of feedback time delay on the stabilization of the unstable equilibrium point of the pendulum. A first attempt on analyzing the effect of time delay on the stability of a planar pendulum was presented in [12], where the author inspired by the PMD (Proportional Minus Delay controller) technique to provide sufficient conditions for the stability of the inverted pendulum when the action of the control input is delayed. Although, this technique is quite simple in implementation, but the analysis becomes more involved when considering highly nonlinear system. In [14] the authors have studied the effect of the time delay on a complete model of the Cart-Pole system. Their approach however relies on a linearization of the motion dynamics around an equilibrium point and the condition for asymptotic stability of the pendulum has been derived using standard results from the stability theory of delayed linear differential equations. Along the same philosophy, a H_∞ approach has been used in [15] to study the behavior of a linear pendulum with input time delay. An important application that involve synchronization of different mechanical systems has been investigated for the case of two inverted pendulums in a master-slave configuration [16]. An appealing feature of this work is the design of the control input that considers time delay in the data transmission.

This paper extends the results from [17] and presents a partial feedback linearization method in combination with a nonlinear state prediction method [18] that anticipates the input time-delay presents in the non-collocated feedback. Specifically, we address the stabilization control problem of the Cart-Pole system and shed lights on the effect of the time delay that arises in the partial feedback technique as a result of the use of networked channel communication. Several coordinate transformations are then processed to transform the Cart-Ploe's dynamics into an upper-triangular form affected with vanishing prediction errors. The presence of these terms can result in finite escape time under standard nested saturation design technique [19]. The control design presented in this paper overcomes these difficulties and employs a combination of the nested saturation and the back-stepping control design to deal with the resulting nonlinearities multiplied by the prediction errors, which can avoid the finite escape time problem in the closed-loop system.

This paper is organized as follows. Section 2 presents the dynamic model of the Cart-Pole system and reminds the basics of partial feedback linearization of the nonlinear actuated equation. The control problem of stabilizing the inverted pendulum subject to a network induced bilateral time delay is then formulated. Section 3, provides preliminaries regarding stability of time delayed functional differential equations along with practical transformation for writing underactuated system in cascaded normal form. In Section 4, the predictor based control scheme is presented. Simulations are given in Section 5 and concluding remarks are presented in Section 6.

2. Problem formulation

2.1. Dynamic equations of the cart-pole system

The Cart-Pendulum system depicted schematically in Fig. 1 consists of a pole mounted on a cart by means of a pivot in such a way that the pole can freely swing in the $x-y$ plane. The equations of motion can be obtained by neglecting friction in the pivot and by applying the Euler-Lagrange formulation. The system model is represented as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = U(t) \tag{1}$$

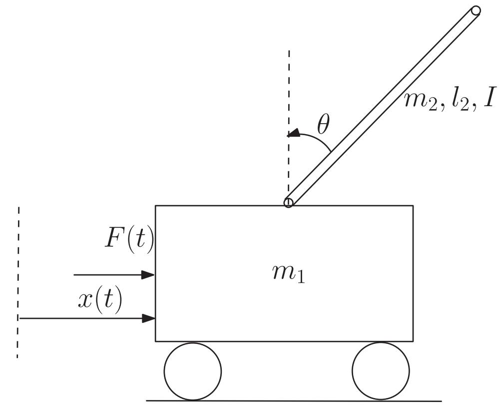

Fig. 1. The Cart-Pole system.

where q, $\dot{q}$ and $\ddot{q}$ denote the position, the velocity and the acceleration of the cart-pole respectively, $M(q)$ is the inertia matrix, $C(q,\dot{q})$ is the centripetal and Coriolis force matrix and $G(q)$ is the gravitational force vector, while $U(t)$ represents the applied force to the system.

For the cart-pole system depicted in Fig. 1 can be described as follows:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} x \\ \theta \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}, \quad \ddot{q} = \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix}$$
$$M(q) = \begin{bmatrix} m_1 + m_2 & m_2 l_2 \cos(\theta) \\ m_2 l_2 \cos(\theta) & I + m_2 l_2^2 \end{bmatrix}, \quad C(q,\dot{q}) = \begin{bmatrix} 0 & m_2 l_2 \dot{\theta} \sin(\theta) \\ 0 & 0 \end{bmatrix}$$
$$G(q) = \begin{bmatrix} 0 \\ -m_2 g l_2 \sin(\theta) \end{bmatrix}, \quad U(t) = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

where m_1 is the mass of the cart, m_2 is the mass of the pendulum, I_2 is the moment of inertia of the pendulum, l_2 is half the length of the pendulum, i.e., the distance from the pivot to the center of mass of the pendulum, g is the acceleration due to gravity and I is the moment of inertia of the pendulum. The cart-pole System has two equilibrium points, one of which is known as the stable vertically downward position where $\theta = \pi$ and the other one being the unstable vertically upward position where $\theta = 0$. $F(t)$ is the horizontal force being applied to the cart to swing the pendulum until it reaches the desired unstable position where $\theta = 0, \dot{\theta} = 0$ and $\dot{x} = 0$.

The dynamics of the inverted pendulum is given as following:

$$\begin{aligned} &(m_1 + m_2)\ddot{x} + m_2 l_2 \cos(\theta)\ddot{\theta} - m_2 l_2 \sin(\theta)\dot{\theta}^2 = F(t) \\ &m_2 l_2 \cos(\theta)\ddot{x} + \left(m_2 l_2^2 + I_2\right)\ddot{\theta} - m_2 g l_2 \sin(\theta) = 0 \end{aligned} \tag{2}$$

It can be observed form the dynamic (2) and its corresponding description, the cart-pole system is an underactuated system having fewer actuators than the number of degree of freedom. Since the cart-pole system is underactuated, partial linearization is needed among others techniques to reduce the design complexity of the controller. After such partial linearization being processed, the inverted pendulum becomes controlled using the dynamic coupling between the systems' variables and thus following the approach proposed in [17], a new control input $u(t)$ is chosen in terms of the cart acceleration is being designed to stabilize the cart–poles' states at the origin. To this end the applied control input $F(t)$ is chosen as follows [17]:

$$\begin{aligned} F(t) = &\left(m_{12}(\theta) - \frac{m_{11}m_{22}}{m_{21}(\theta)}\right)u(t) + (m_1 + m_2)g\tan(\theta) \\ &- m_2 l_2 \sin(\theta)\dot{\theta}^2 \end{aligned} \tag{3}$$