ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Move Suppression Calculations for Well-Conditioned MPC

Michael Short

School of Science & Engineering, Teesside University, Middlesbrough TS1 3BA, UK

ARTICLE INFO

Article history:
Received 23 July 2015
Received in revised form
29 June 2016
Accepted 29 November 2016

Keywords:
Predictive Control
Generalized Predictive Control
Dynamic Matrix Control
Move Suppression
Numerical Conditioning

ABSTRACT

Several popular tuning strategies applicable to Model Predictive Control (MPC) schemes such as GPC and DMC have previously been developed. Many of these tuning strategies require an approximate model of the controlled process to be obtained, typically of the First Order Plus Dead Time type. One popular method uses such a model to analytically calculate an approximate value of the move suppression coefficient to achieve a desired condition number for the regularized system dynamic matrix; however it is not always accurate and tends to under-estimate the required value. In this paper an off-line method is presented to exactly calculate the move suppression coefficient required to achieve a desired condition number directly from the unregularized system dynamic matrix. This method involves an Eigendecomposition of the system dynamic matrix - which may be too unwieldy in some cases –and a simpler analytical expression is also derived. This analytical expression provides a guaranteed tight upper bound on the required move suppression coefficient yielding a tuning formula which is easy to apply, even in on-line situations. Both methods do not require the use of approximate or reduced order process models for their application. Simulation examples and perturbation studies illustrate the effectiveness of the methods in both off-line and on-line MPC configurations. It is shown that accurate conditioning and improved closed loop robustness can be achieved.

1. Introduction

Model predictive control (MPC) has been implemented widely in the chemical and process control industries since its introduction in the 1970s [1–3]. Although a large number of different approaches to MPC have been formulated, two of the most popular with practitioners have proved to be the Dynamic Matrix Control (DMC) technique [4] and the Generalized Predictive Control (GPC) technique [5]. Both schemes employ a receding-horizon approach which minimizes, at each time step, a multi-stage quadratic cost function involving the predicted future errors and weighted magnitude of the applied incremental control moves. A typical MPC controller has many tuneable parameters: aside from considerations regarding the process parameterisation, the principal ones of interest for DMC and GPC are the choice of sampling time T, the length of the prediction horizon P and the control horizon M, and also the value of the move suppression coefficient λ. The latter parameter applies a weight on the magnitude of the projected control moves in the objective function. Due to the complex relationships between these tuneable parameters and the closed loop system properties, many previous authors have suggested ‘tuning rules’ that allow a user to configure an MPC instance to achieve a desired level of closed-loop performance [3–14]. This paper is concerned with the selection of the move suppression coefficient, which serves a dual purpose of conditioning the system matrix before its inversion and suppressing aggressive control actions [3–14]. This non-negative dimensionless parameter is known to have a significant impact upon performance and robustness [4–8], and in practice proves difficult to tune empirically (even for experienced control engineers) as recent work has highlighted [12].

A variety of methods have been proposed to tune this parameter. In [11] the authors describe a procedure for iteratively tuning λ for a GPC controller, assuming a Second Order Plus Dead Time (SOPDT) process model. The chosen performance criteria are that the closed loop poles satisfy certain bounds; at each iteration of the search, fourth order polynomials are solved and the GPC gains recomputed from quadratic formulae. In [12], the authors propose to use the Nelder-Mead downhill simplex algorithm to search for values of λ which minimize an objective criteria in multivariable DMC controllers. The objective criteria that the authors suggest are based upon the magnitude and shape of the Manipulated Variables (MVs). It is suggested that one of the ways that the aggressiveness of the controller is measured is by the percentage of overshoot that occurs in each of the MVs following a step setpoint change in reference; it is recommended that the search aims to produce tunings with no more than 20% MV overshoot following a step on any reference. This choice of metric is limited to input/output relationships of Type 0, i.e. those which are self-regulating, as the presence of one or more open-loop integrators renders the metric undefined. The authors in [9] describe an analytical method to compute the required move suppression

http://dx.doi.org/10.1016/j.isatra.2016.11.020
0019-0578/

coefficients to achieve a pre-specified closed loop performance for FOPDT processes when the control horizon M is equal to either 1 or 2. The method has also been extended to multivariable processes which can be approximated as a matrix of FOPDT responses [10]. No considerations of numerical stability or smoothness of control actions are considered in either method; extensions to higher order system models and/or larger control horizons was not considered in these works.

One method which has proved popular for calculation of the move supression parameter was previously proposed by Shridhar and Cooper [6]. A key contribution of this work was the identification of the link between the condition number of the regularized Gramian of the system dynamic matrix and the closed-loop performance and robustness of the resulting controller. For its application, the method requires an approximate First Order Plus Dead Time (FOPDT) model of the controlled process to be obtained. This model is employed for a number of reasons, principally to analytically calculate from its parameters an approximate value of the move suppression coefficient to achieve a certain condition number C for the regularized Gramian. The aim of this procedure is to ensure that 'the condition number is always bounded by a fixed low value' [6]. The tuning strategy, although principally developed for DMC, can also be applied to control of FOPDT models with GPC [6] and has been extended to multivariable MPC [7] and integrating processes [8]. However, the requirement for low-order approximate process models renders the method only suited to off-line MPC tuning for processes in which such a model is reasonable. In addition, as will be demonstrated through examples in a later section, the accuracy of the achieved condition number is highly dependent upon the accuracy and validity of the low-order approximation; oftentimes the required move suppression is underestimated, and the resulting condition number exceeds that which is desired (in some cases by $\approx 100\%$).[1] This warrants the consideration of possible alternative methods to calculate the move suppression coefficient directly from the employed process model (i.e. avoid the need to utilize an approximate low-order model) in order to achieve well-conditioned MPC [16].

In this paper, two methods are presented for this purpose. The first method exactly calculates the required move suppression needed to achieve a given conditioning directly from the unregularized system matrix. This method requires an Eigen decomposition of the system matrix and is suitable for an off-line MPC design using a suitable numerical software package. The second method involves a simple analytical expression using only the trace of the unregularized matrix and its square to obtain a tight upper bound on the required value of move suppression, and is easy to apply. Simulation results and perturbation studies verify that accurate conditioning and improved closed loop robustness can be achieved. Before describing the proposed methods, it must be stressed that the use of move suppression (regularization) is not the only possible method to have been proposed to improve the numerical properties and robustness of MPC algorithms. Methods based upon Principal Components Analysis (PCA) [15] or the use of a 'shifted' DMC algorithm [16] can both achieve such goals and correct rank deficiencies in MPC controllers without the need to obtain a reduced order approximation of the actual progress dynamics [16]. However in this paper, the focus will be upon move suppression only, and specifically improvements to the methods proposed in the works [6–8].

The remainder of the paper is structured as follows. Section 2 introduces preliminaries. The methods to calculate the move suppression parameter are presented in Section 3 of the paper. Section 4 presents simulation examples, perturbation studies and analysis to illustrate the effectiveness of the proposed methods in on-line and off-line situations. A short conclusion is given in Section 5.

[1] Although this is in line with observations made in [14] (also cited in [12]), it must be cautioned that the results presented in [14] do not seem reproducible and the extent of the problem significantly over-estimated in this paper: please refer to Appendix A for details.

2. Preliminaries

MPC algorithms employ a receding-horizon optimization of the process input which minimizes a multi-stage quadratic cost function at each time step. In the Single-Input-Single-Output (SISO) case the cost function to be minimized at each discrete time step t can typically be written as:

$$J = \sum_{i=1}^{P} (\hat{y}(t+d+i|t) - r(t+i))^2 + \lambda \sum_{j=1}^{M} (\Delta u(t+j-1)^2 \tag{1}$$

In which $\hat{y}(t+k|t)$ is a k-step ahead prediction of the process output at time step t, $d \geq 0$ represents the integer part of the system time delay (excluding the zero-order hold), $r(k)$ is the value of a known reference/setpoint sequence at step k, and the decision variables $\Delta u(k)$ represent the incremental change in the applied controls at step k. The integer parameter $M > 0$ represents the length of a short future control horizon, while the integer $P \geq M$ represents the length of the prediction horizon. The move suppression parameter $\lambda \geq 0$ introduces an additional weighted quadratic penalty on the magnitude of the control moves into the objective function. The principal difference between approaches such as DMC and GPC lies in the assumptions made of the process and disturbance model, and in the techniques employed to obtain the process predictions; the latter also allows more flexibility in the choice of weights than that shown in (1) - although (1) represents its typical 'default' configuration for industrial plant [5]. In the unconstrained case minimization of (1) leads to an analytical expression for calculating the projected optimal control moves at each time step, which surmounts to solving the system of linear equations $(G^TG+\lambda I)\Delta u(t)=G^Te(t)$, where the M-vector $\Delta u(t)$ represents the optimal control moves to make at time t, G is the system dynamic matrix (of dimension P-by-M) consisting of the shifted system step co-efficients arranged in Toeplitz fashion. The vector $e(t)$ represents, at time t, the future (predicted) errors between the free trajectory of the process and the desired trajectory along each step of the prediction horizon P. The move suppression parameter λ appears in the solution as a regularization parameter applied to G^TG, the Gramian of the system matrix. As MPC is a receding-horizon control, only the first element of the M-vector of optimal controls $\Delta u(t)$ is applied at time t. At time step $t+1$, the optimization is repeated to obtain $\Delta u(t+1)$ using the newly acquired knowledge of the plant state and an updated set of predicted errors $e(t+1)$. This process repeats indefinitely. For the remainder of the current work, the focus is upon SISO processes, with the understanding that the results also generalize to Multi-Input-Multi-Output (MIMO) systems. This generalization is possible since for any MIMO system, it is possible to partition the dynamic matrix into sub-blocks connecting each input-output pair [2,7]; the proposed SISO method may then be applied to each such sub-block in turn.

3. Move suppression calculations

3.1. Conditioning and robustness

Typically, for the implementation of MPC one desires the first row of the left pseudo-inverse 'gain' matrix $G^+=(G^TG+\lambda I)^{-1}G^T$ [2]. To obtain this gain vector, the inverse of the matrix $(G^TG+\lambda I)^{-1}$ is required; standard numerical techniques such as Gauss-Jordan elimination can be employed to obtain it [2,17]. Let the unregularized dynamic matrix G^TG be denoted by A, with the

Please cite this article as: Short M. Move Suppression Calculations for Well-Conditioned MPC. ISA Transactions (2016), http://dx.doi.org/10.1016/j.isatra.2016.11.020