ISA Transactions 63 (2016) 1–10

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Observation and sliding mode observer for nonlinear fractional-order system with unknown input

Nadia Djeghali [a], Said Djennoune [a,*], Maamar Bettayeb [b,c], Malek Ghanes [d], Jean-Pierre Barbot [d]

[a] Laboratoire de Conception et Conduite des Systèmes de Production, University Mouloud Mammeri, Tizi-Ouzou, Algeria
[b] Department of Electrical & Computer Engineering, University of Sharjah, United Arab Emirates
[c] Distinguished Adjoint Professor at Center of Excellence in Intelligent Engineering Systems (CEIES), King Abdulaziz University, Jeddah, Saudi Arabia
[d] ECS-Lab, ENSEA, 6 avenue du Ponceau, 95014 Cergy-Pontoise, France

ARTICLE INFO

Article history:
Received 15 April 2015
Received in revised form
29 October 2015
Accepted 18 February 2016
Available online 5 March 2016
This paper was recommended for publication by Dr. Mohammad Haeri

Keywords:
Nonlinear fractional-order systems
Observability
Left invertibility
Sliding mode observers
Fault detection and estimation
Lyapunov stability

ABSTRACT

The main purpose of this paper is twofold. First, the observability and the left invertibility properties and the observable canonical form for nonlinear fractional-order systems are introduced. By using a transformation, we show that these properties can be deduced from an equivalent nonlinear integer-order system. Second, a step by step sliding mode observer for fault detection and estimation in nonlinear fractional-order systems is proposed. Starting with a chained fractional-order integrators form, a step by step first-order sliding mode observer is designed. The finite time convergence of the observer is established by using Lyapunov stability theory. A numerical example is given to illustrate the performance of the proposed approach.

1. Introduction

The sliding mode technique has been recognized as a powerful tool to design robust controllers for linear and nonlinear systems [1–4]. This technique has also found a great success in the design of observers for linear and nonlinear dynamical systems with unknown input. These observers have been applied to different problems such as the detection and estimation of actuator and sensor faults [5–8]. Two kinds of observers with unknown input should be distinguished. In the first kind, the unknown input is considered as a disturbance and must be rejected so that it does not affect the estimation of state variables. The second kind is applied to the diagnosis and fault detection problems where the unknown input is considered as the fault signal. Then, in this case, not only state variables are estimated but even the unknown input is reconstructed.

In recent years, fractional-order systems received a growing interest and have enjoyed a wide success in modeling physical phenomena and in the synthesis of robust controllers. Several books, [9–13], provide a good source of references on fractional calculus and its applications.

In control theory, many studies extend the most well known control and observer design methods in the classical integer-order modeling to fractional-order systems. For linear systems, in the frequency domain, a fractional order Tilt Integral Derivative (TID) controller [14] and the well known CRONE (Commande Robuste Ordre Non Entier) controllers [15] have been developed. Fractional-order PID controllers [16] and fractional lead-lag compensators [17] were proposed. Other well-known control strategies designed for integer-order systems have been extended to fractional-order systems. State space fractional design methods based on pole placement are developed in [18]. State estimators-based observer design for fractional-order systems is also intensively considered in the literature. Synchronization based observers for nonlinear fractional-order chaotic systems is treated in [19,20]. In [21], full-order observer design for a class of nonlinear fractional-order systems is proposed. Observer with unknown input for linear fractional-order systems is designed in [22].

The sliding mode control (SMC) technique has been also extended to fractional-order systems [23–29]. A sliding surface

* Corresponding author.
E-mail addresses: djeghali_nadya@yahoo.fr (N. Djeghali), s_djennoune@yahoo.fr (S. Djennoune), maamar@sharjah.ac.ae (M. Bettayeb), Malek.Ghanes@ensea.fr (M. Ghanes), barbot@ensea.fr (J.-P. Barbot).

http://dx.doi.org/10.1016/j.isatra.2016.02.015
0019-0578/

defined on a manifold expressed with fractional-order integral is introduced in [25,28,27]. In [26,29], the authors used LMI techniques to design a sliding mode controller for nonlinear systems. Higher order sliding mode controllers for multivariable systems were developed in [27]. In [30], the problem of state estimation for a class of uncertain nonlinear fractional-order systems, using sliding mode technique, is investigated. A sliding mode observer for fault estimation in linear fractional-order systems is addressed in [31]. In the design of observers for nonlinear integer-order systems subject to unknown inputs, the system is usually associated to the canonical observable form [32–34]. If the nonlinear integer-order system fulfills the observability property, then, the canonical observable form can be constructed by a change of coordinates. Furthermore, if the nonlinear fractional-order system satisfies the left invertibility condition, then the unknown input can be estimated from the knowledge of the outputs. To the best of our knowledge, for nonlinear fractional-order systems, some features such as observability, left invertibility and normal forms have not been addressed in the literature. The first contribution of the present paper is to give a key direction to study these important properties. The second contribution is to develop a step by step sliding mode observer to fault estimation in nonlinear fractional-order systems. The main advantage of this observer is the finite-time convergence.

The rest of the paper is organized as follows.

In Section 2, definitions and some properties of fractional-order derivatives and fractional-order systems are recalled. In Section 3, the observability, left invertibility and canonical observation form features for nonlinear fractional-order systems are introduced. In Section 4, the design model addressed throughout this paper is described and the main result on the convergence of the proposed step by step sliding mode observer is developed. In Section 5, a numerical example is given to illustrate the performance of the proposed observer.

2. Preliminary definitions

Let $L_1[a\ b]$ denote the space of Lebesgue integrable real-valued functions $f(t)$ of the variable t, which represents the time, on the interval $[a\ b]$, $a, b \in \Re_+$, such that $0 \le a < b < \infty$ and $\Re_+$ denotes the non-negative real numbers set. Let $C[a\ b]$ be the space of functions $f(t)$ which are continuous on $[a\ b]$ and we denote by C^k the space of real-valued functions $f(t)$ which have continuous derivatives up to order $k-1$ such that $f^{(k-1)}(t) \in C[a\ b]$ where $f^{(i)}(t)$ is the i-th integer-order derivative of $f(t)$.

Definition 1 (*Kilbas et al.* [9]). Let $f(t) \in L_1[a\ b]$ be a function of the variable t, $t \in [a\ b]$. The fractional integral of order $\alpha \in \Re_+$ is defined by the Riemann–Liouville integral

$$ {}_aI_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau \tag{1}$$

where Euler's Gamma function $\Gamma(\alpha)$ is defined as

$$\Gamma(\alpha) = \int_0^{\infty} \nu^{\alpha-1} e^{-\nu}\, d\nu \tag{2}$$

Let α be a real number such that $m-1 < \alpha \le m$, where m is an integer number. Two main definitions of fractional-order derivatives are introduced in the literature, namely Riemann–Liouville's derivative, and Caputo's derivative [9].

Definition 2 (*Kilbas et al.* [9]). The Riemann–Liouville fractional derivative of order α of $f(t) \in C^m[a\ b]$; $t \in [a\ b]$, is defined as

$$ {}_a^{\mathrm{RL}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_a^t (t-\tau)^{m-\alpha-1} f(\tau)\, d\tau \tag{3}$$

Definition 3 (*Kilbas et al.* [9]). Caputo's derivative of order α of $f(t) \in C^m[a\ b]$; $t \in [a\ b]$, is defined as

$$ {}_a^{\mathrm{C}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t (t-\tau)^{m-\alpha-1} \frac{d^m f(\tau)}{d\tau^m}\, d\tau \tag{4}$$

Even if the Riemann–Liouville derivative reflects more precisely the typical long memory behavior of fractional-order systems, this derivative definition has a major limitation that the initial conditions are in terms of fractional-order derivatives of the variable. In contrast, the definition given by Caputo's derivative needs the initial conditions which are specified in the usual way. For $0 < \alpha < 1$, the following relation between the Riemann–Liouville and the Caputo derivatives is established [9].

$$ {}_a^{\mathrm{C}}D_t^{\alpha} f(t) = {}_a^{\mathrm{RL}}D_t^{\alpha} (f(t) - f(a)) \tag{5}$$

In this paper, Caputo's fractional-order derivative is used. The origin of time is taken zero. For simplicity, we use the following notations:

$$\begin{aligned} & I_0^{\alpha} f(t) \triangleq I^{\alpha} f(t) \\ & {}_0^{\mathrm{C}}D_t^{\alpha} f(t) \triangleq D^{\alpha} f(t) \\ & \frac{df(t)}{dt} \triangleq Df(t) \end{aligned} \tag{6}$$

Some useful properties of the fractional integral and derivative are summarized below.

Proposition 1 (*Kilbas et al.* [9]). *Let $f(t)$ be a real-valued function of the variable $t \in [a\ b] \subset \Re_+$, then*

1. $I^0 f(t) = f(t)$ *and* $I^{\alpha} I^{\beta} f(t) = I^{\alpha+\beta} f(t), f(t) \in L_1[a\ b], \forall \alpha, \beta \in \Re_+$
2. *Caputo's derivation operator is linear, that is*:
$$D^{\alpha}(c_1 f_1(t) + c_2 f_2(t)) = c_1 D^{\alpha} f_1(t) + c_2 D^{\alpha} f_2(t) \tag{7}$$
where c_1 and c_2 are any real numbers.
3. *Let C be a constant, then $D^{\alpha} C = 0$*
4. *The fractional-order integral and Caputo's derivative of $f(t) = t^{\beta}$ are given by*
$$I^{\alpha} t^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\alpha+1)} t^{\beta+\alpha} \tag{8a}$$
$$D^{\alpha} t^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} t^{\beta-\alpha} \tag{8b}$$

3. Observability, left invertibility and observation form for nonlinear fractional-order systems

A nonlinear fractional-order time-invariant system is described by the state-space representation which involves fractional derivatives of state variables. Consider the class of the single input, single output nonlinear system described by

$$D^{[\alpha]} x(t) = f(x(t)) + g(x(t)) u(t) \tag{9a}$$

$$y(t) = h(x(t)) \tag{9b}$$

where $x(t) = [x_1\ x_2 \ldots x_n]^T \in \mathcal{M} \subset \Re^n$ denotes the n-dimensional state vector, $u(t) \in \Re$ is the input control, $y(t) \in \Re$ is the output and $D^{[\alpha]} = \left[D^{\alpha_1}\ D^{\alpha_2} \ldots D^{\alpha_n}\right]^T$ is the fractional differentiation vector operator of orders $\alpha_i \in \Re_+, i = 1, 2, \ldots, n$. If all orders α_i are equal,