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Improved stability and stabilization design for networked control systems using new quadruple-integral functionals

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ABSTRACT

This paper investigates stability analysis and stabilization for networked control systems. By a refined delay decomposition approach, slightly different Lyapunov–Krasovskii functionals (LKFs) with quadruple-integral terms and augmented vectors containing triple-integral forms of state are constructed. New integral inequalities are proposed to estimate the cross terms from derivatives of the LKFs, which can be proved to offer tighter bounds than what the Jensen one produces theoretically. Moreover, the non-strictly proper rational functions in deriving process are fully handled via reciprocally convex approach. A state feedback controller design approach is also developed. Numerical examples and applications to practical power and oscillator systems demonstrate the superiority of the proposed criteria in conservatism reduction compared to some existing ones.

1. Introduction

Control systems with different components such as sensors, controllers and actuators connected through shared communication networks are called networked control systems (NCSs) [1]. Compared with traditional control systems, NCSs offer high reliability, increased flexibility of systems and reduced weight and cost. Thus, in the last decade, network-based control strategy has found successful applications in the wide fields of science and engineering. However, by the insertions of communication channels, network phenomena including network -induced delay and data packet dropouts will be unavoidably encountered, which may lead to performance degradation or even instability [2]. On the other hand, stability and stabilization are central issues to study the behavior of a system, which have received much attention [3–6]. Therefore, research on stability and stabilization for NCSs has significantly theoretical and practical values.

The input delay approach has been widely used in sampled-data control systems since it is easy to deal with aperiodic sampling action [7,8]. As a special case of sampled-data control system, NCS can also be modeled by such an approach, where the delay and packet dropouts were absorbed into a state delay [9–13]. Due to limited bandwidth, the network phenomena are usually time-varying. Therefore, this kind of model is very close to real NCSs and allows the active developments of time-delay systems to study NCSs. However, since the delay and packet dropouts differ in properties (such as lower and upper bounds etc.) and their impacts on system performance are also different, it is unreasonable to simply lump them together [10].

Stability criteria are usually obtained by Lyapunov theory, and introduce some inevitable conservatism, which is usually indexed by the derived maximum upper delay bound (MUDB) [14]. In [15], a novel stability condition is derived in consideration of tradeoff between conservatism and computational complexity via several new techniques, the contributions of which are analyzed one by one based on theory analysis, and useful guidelines for improving criterion are discussed. According to [15], two research directions have been recognized as efficient methods to reduce conservatism. One is the constructions of appropriate LKFs. Augmented LKFs comprising of triple-integral terms were utilized in [16,17], whose merits have been discussed thoroughly in [18]. Furthermore, adding some quadruple-integral forms of LKFs is beneficial for reduction of conservatism such as reported in [19,20]. By dividing the delay interval into multiple equidistant subintervals, authors [21,22] constructed different LKFs on these subintervals and it can be expected that more subintervals lead to less conservatism. However, the conservatism reduction tends to be inapparent as increasing of delay decomposition number, which yields more computational burden [14]. Inspired by the optimal delay

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decomposition idea, a parameter varying within the delay range was used in [23–25] to achieve two unequal segments. Moreover, in [25], the interval $[0,\ \eta_l]$ (η_l is the lower bound of delay) was split into two uniform segments to reduce conservatism introduced by larger lower bounds.

The other prevalent direction to reduce conservatism is the more accurate estimation of the cross terms when calculating the derivative of LKF. In [14], it has been found that slack matrices are effective to relax cross terms and it has been proved that four types of free-weighting matrix methods are equivalent to each other. On the other hand, Jensen inequality has played a key role in this trend even if it is at the price of considerable conservatism. Recently, Wirtinger inequality was reported in [26] to overcome the conservatism. In [27], the free-matrix-based inequality was developed to provide freedom in reducing conservatism. In [28], less conservatism is obtained by using free matrices to handle relationships among terms $x(\alpha), x(\beta), \int_\alpha^\beta x(s)ds$ and $\int_\alpha^\beta \int_\alpha^s x(u)duds$. Taking advantages of Legendre polynomials, the Bessel–Legendre (B–L) inequality was performed in [29], which encompassed the Jensen and Wirtinger ones as particular cases. Moreover, much effort has been devoted to treat double-integral terms obtained from derivatives of triple-integral functionals. New integral inequalities were developed via intermediate terms in [30], which turned into the inequalities in [26,29] by choosing suitable auxiliary functions. An extension of Wirtinger-based inequality was derived in [31] to offer tighter bound of double-integral form, which has been only applied to constant delays. In [32], by a constructive modification, some Jensen inequalities are refined to provide larger delay bounds. However, [26,29] and [30–32] have just dealt with single and double-integrals, respectively, while triple-integral terms should also be estimated if quadruple- integral forms of LKFs are introduced. Therefore, there still exists much room for improvement.

This paper proposes new stability analysis and stabilization approaches for NCSs with network phenomena. The contributions to reduce conservatism of this paper can be generalized as follows: (i) a refined delay decomposition approach is developed to split delay interval into two unequal subintervals such that the impacts of network phenomena are distinguished. In addition, the interval $[0,\eta_l]$ (η_l is the lower bound of delay) is decomposed into λ uniform segments to reduce conservatism for larger lower bounds. (ii) New LKFs with quadruple-integral terms and augmented vectors including triple-integral forms are established to make full use of information on delay range. (iii) For further reduction of conservatism, the integral terms in the derivatives of LKFs are estimated more exactly via new integral inequalities combined with reciprocally convex approach. Due to the above treatments, the derived stability criteria and stabilization approach are less conservative and more computationally attractive than some existing literature.

The rest of the paper is organized as follows. The problems are formulated and the new integral inequalities are presented in Section 2. Section 3 introduces stability criteria for the NCSs in terms of linear matrix inequalities (LMIs). In Section 4, a feedback controller design approach is also developed. Section 5 gives two numerical examples and applications to practical power and oscillator systems to demonstrate the outperformance of the proposed approaches compared with some existing ones. Finally, Section 6 draws the conclusion.

1.1. Notations

$\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{m\times n}$ means the set of all $m\times n$ real matrices. The superscripts '-1' and 'T' denote the inverse and transpose of a matrix, respectively; $X>0$ ($X\geq 0$) means X is symmetric positive definite (positive semi-definite). $*$ stands for symmetric block in the symmetric matrices; $diag\{\cdot\}$ exhibits a block diagonal matrix. $sym\{P\}=P+P^T$. n is the dimension of the system. I and 0 are identity and zero matrices of appropriate dimensions, respectively. e_i are block entry matrices, for example, $e_2^T=[0\ I\ 0\ldots 0]$. δ is the maximum number of consecutive packet dropouts. τ_l and τ_u are the lower and upper bounds of network-induced delay, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The formulae are numbered consecutively and the numbers of formulae are valid throughout the paper.

2. Preliminaries

In NCS, the plant is a linear continuous-time system described by the following dynamic system model:

$$\dot{x}(t)=Ax(t)+Bu(t), \tag{1}$$

where $x(t)\in\Re^n$ and $u(t)\in\Re^m$ are state and control input vectors, respectively. $A\in\Re^{n\times n}$ and $B\in\Re^{n\times m}$ are constant matrices with appropriate dimensions.

Considering the NCS configuration illustrated in Fig. 1, the control input $u(t)$ realized by a zero-order holder (ZOH) is a piecewise constant function. Suppose that the sensor is clock-driven and the controller, actuator and ZOH are event-driven. The sampling period is assumed to be a positive constant h. The sampling instants of data transmitted from the sensor to the ZOH successfully are denoted by the set $\{i_1h, i_2h, \ldots, i_kh\}$, where i_k are positive integers. The number of consecutive packet dropouts is denoted by $\delta_k=i_{k+1}-i_k-1$. Since the control gain is a constant, the backward delay τ_{sc_k} and forward delay τ_{ca_k} can be combined as $\tau_k=\tau_{sc_k}+\tau_{ca_k}$. The network-induced delay and the number of consecutive data packet dropouts are bounded with $\tau_k\in[\tau_l,\ \tau_u]$ and $\delta_k\in[0,\ \delta]$, respectively. Then, the NCS can be modeled as

$$\begin{aligned}&\dot{x}(t)=Ax(t)+Bu(t), \quad t\in[i_kh+\tau_k, \quad i_{k+1}h+\tau_{k+1}),\\ &\quad u(t^+)=Kx(t-\tau_k), \quad t\in\{i_kh+\tau_k, \quad k=1,2,\ldots\},\end{aligned} \tag{2}$$

where K is the state feedback gain matrix. By defining $\eta(t)=t-i_kh$, the NCS takes the following form

$$\dot{x}(t)=Ax(t)+A_dx(t-\eta(t)), \quad t\in[i_kh+\tau_k, i_{k+1}h+\tau_{k+1}). \tag{3}$$

where $A_d=BK$ and $\eta(t)$ is time-varying delay satisfying

$$\tau_l\leq\eta(t)\leq\tau_u+(\delta+1)h. \tag{4}$$

Remark 1. With the rapid development of networked control technology, stability and stabilization for NCS with network phenomena have received much attention. NCS can be viewed as a special case of time-delay system or sampled-data system [11]. Following the same modeling procedure in [9–13], the delay and packet dropouts are taken into consideration by the input delay $\eta(t)$ without loss of generality. For a time-delay system, it is of great significance to derive MUDB to guarantee the stability of system. Moreover, if the delay τ_k is assumed to be zero, the system

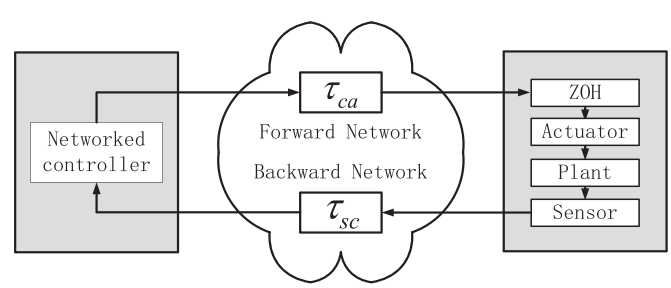

Fig. 1. Typical configuration of networked control system.