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Performance analysis of fractional order extremum seeking control

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ABSTRACT

Extremum-seeking scheme is a powerful adaptive technique to optimize steady-state system performance. In this paper, a novel extremum-seeking scheme for the optimization of nonlinear plants using fractional order calculus is proposed. The fractional order extremum-seeking algorithm only utilizes output measurements of the plant, however, it performs superior in many aspects such as convergence speed and robustness. A detailed stability analysis is given to not only guarantee a faster convergence of the system to an adjustable neighborhood of the optimum but also confirm a better robustness for proposed algorithm. Furthermore, simulation and experimental results demonstrate that the fractional order extremum-seeking scheme for nonlinear systems outperforms the traditional integer order one.

1. Introduction

Extremum seeking control (ESC) is an online adaptive optimization algorithm attempting to determine the extremum value of an unknown nonlinear performance function in real-time. Thereby, it reduces the optimization downtime by eliminating the need for offline data analysis. This optimization method was successfully applied to a wide range of engineering applications including maximum power point tracking in renewable energy systems [1–6], control of ABS brakes [7–9], combustion engine timing control [10,11], mobile robots path planing [12–15] and so on. A descriptive survey on extremum seeking control and its applications can be found in [16,17].

Due to the wide range of engineering applications, there exist a growing interest among researchers and scientists to improve the performance and reliability of this algorithm by providing better tuning and calibration methods. Owing to these Scientific efforts, there have been a number of developments in ESC algorithm including digital implementation of sinusoidal perturbed ESC, the use of periodic non-sinusoidal perturbation signals, the use of a time-dependent dither signal amplitude and the development of stochastic perturbed ESC [18,19]. In all the proposed algorithms, the convergence speed of ESC scheme is proved to be proportional to the second order derivative of output of unknown nonlinear performance function with respect to its input ($\ddot{y} = \frac{d^2 y = f(\theta)}{d\theta^2}$). This dependency may result in destabilization of the system when variations in $\ddot{y}$ is steep due to the plant condition variations [18]. To reduce this dependency, different approaches have been proposed in the literature.

One of the traditional approaches is to compensate $\ddot{y}$ variations by employing additional compensator(s) in the ESC scheme [20]. The transient response of the averaged-linearized model of this ESC can be adjusted by adequate tuning of compensator(s). Although this is an effective method to eliminate the issue when $\ddot{y}$ variations are small, since these variations are unknown, an estimation approach is required to predict the $\ddot{y}$ changes ahead of the time and tune the compensator coefficients accordingly. Nesic in [21] has presented an ESC tuning guidelines which ensures larger domain of attraction and faster convergence speed for extremum seeking algorithm. It aught to be mentioned that by properly tuning ESC parameters, the global peak can be achieved in the presence of local extremum(s), as it is claimed in [21]. The same issue has been investigated by Tan et al. by analyzing various periodic perturbation signals [19]. In other works, additional loops (i.e. Newton-based ESC) and/or complex mathematical blocks (i.e. Lyapunov-based ESC) have been employed in order to not only reduce the convergence speed but also enhance the performance of the system [3,22–24,14,25–30].

In this paper, to improve the transient response of ESC, a novel extremum-seeking scheme for the optimization of nonlinear plants utilizing fractional order calculus is proposed. A detailed stability analysis of this fractional order extremum seeking control (FO-ESC) scheme is provided to guarantee the convergence of the system to an adjustable neighborhood of the optimum. Furthermore, as will be demonstrated, special features of fractional order operators, such as "locality" and "generalized stability criteria" improve the most important criteria for extremum seeking schemes: convergence speed and robustness.

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Unlike other proposed ESC schemes in the literature, neither additional feed-back/feed-forward loops nor complex mathematical calculations (e.g. matrix calculation) is required for the proposed FO-ESC in order to improve the performance and convergence speed of the system. This feature simplifies the implementation of FO-ESC and reduces the calculation time and implementation cost.

In order to compare FO-ESC and IO-ESC, an averaged linearized model of FO-ESC is derived and analyzed against the equivalent averaged linearized model for IO-ESC. Comparing these two models illustrates the aforementioned advantages of fractional order operators utilization in the ESC.

The rest of the paper is organized as follows: Section 2 introduces the fractional order calculus. Section 3 presents an overview of extremum seeking algorithm. In Sections 4 and 5, the proposed FO-ESC is introduced and the stability of its averaged model is investigated. FO-ESC analysis and comparison with IO-ESC as well as simulation and experimental results are presented in Section 6. Concluding remarks are presented in Section 7.

2. Fractional order derivatives and integral definitions

The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with letter between Leibniz and L'Hospital in 1695 [31].

Nowadays, two popular definitions are used for the general fractional differintegral; Riemann–Liouville (RL) definition and Caputo definition [32]. The RL derivative is defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau, \tag{1} $$

for $(n-1<\alpha<n)$ and $\Gamma(.)$ is the *Gamma* function. When $a=0$, sometime authors use $\frac{d^\alpha}{dt^\alpha}$ notation which is equal to ${}_0D_t^\alpha$.

The Caputo derivative is defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau. \tag{2} $$

In contrast to the RL fractional derivative, when solving differential equations using Caputo's definition, it is not necessary to define the fractional order initial conditions. More details regarding these two definitions and their differences have been discussed in [32]. Fractional derivatives and integrals properties and applications can be found in [33–35,31,36].

Since introduction of fractional calculus to engineering world, the modeling of physical phenomena using fractional order operators and fractional order controllers have been widely investigated among researchers and scientist in this field. All previous researches on the application of fractional order operators in the engineering field imply the superiority of the fractional order operators compared to the classical integer order ones from the view point of robustness and performance [37,38,31,39,34,32,46,47,48].

3. Extremum seeking algorithm

In recent years, various types of ESC structures have been introduced and investigated in the literature and among different algorithms, sinusoidal perturbed ESC structure has drown the most interest among researchers [17]. The general form of a single input–single output periodic perturbed extremum seeking scheme is shown in Fig. 1. As shown in this figure, this type of ESC employs a slow periodic perturbation, $\sin(\omega t)$, and adds it to the estimated signal $\hat{\theta}$. Because of the slow dynamics of the perturbation signal, the plant appears as a static map $(y=f(\theta))$ to ESC and its dynamics does not interfere with the extremum seeking scheme [40]. If the estimated signal, $\hat{\theta}$, is on either side of the extremum point, θ^*, the perturbation, $a\sin(\omega t)$, creates a periodic response of y which is either in phase or out of phase with $a\sin(\omega t)$. The high-pass filter eliminates the "DC component" of y. Thus perturbation signal, $a\sin(\omega t)$, and output signals are two approximately sinusoidal waveform which are in phase if $\hat{\theta}<\theta^*$ or out of phase if $\hat{\theta}>\theta^*$ [40].

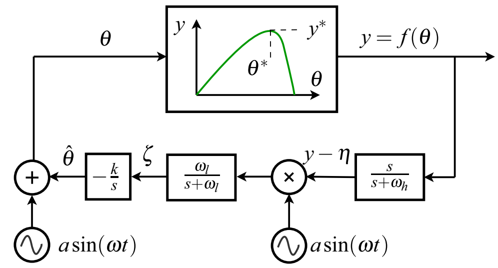

Fig. 1. Block diagram of a single input - single output periodic perturbed extremum seeking algorithm.

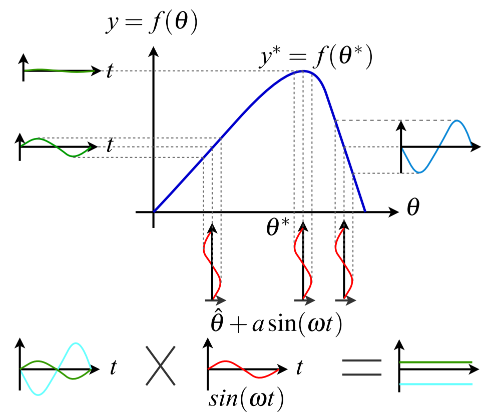

Fig. 2. Extremum seeking algorithm operation for a nonlinear static plant [41].

Fig. 2 illustrates the ESC operation for a nonlinear static plant ,$(y=f(\theta))$ as described above. In this figure, the output of ESC algorithm has been depicted when the operating point is larger, equal or smaller than extremum point. Since product of two in phase signals gives a signal with a positive mean and this product results a negative mean for two out of phase signal, this feature can be used to find the operating point using a gradient detector [41].

The mathematical model for ESC scheme (Fig. 1) can be written as

$$ \begin{cases} y = f(\theta) \\ \dot{\hat{\theta}} = k\zeta \\ \dot{\zeta} = -\omega_l\zeta + \omega_l(y-\eta)a\sin(\omega t) \\ \dot{\eta} = -\omega_h\eta + \omega_h y \end{cases}. \tag{3} $$

To obtain the optimal performance of ESC loop, perturbation frequency, ω, amplitude, a, gradient update law gain, k, and filter cut-off frequencies, ω_h and ω_l, must be calibrated adequately. Some general rules have been listed in the literature as ESC design rules [25,42]. To ensure that the plant dynamics will not be captured by ESC loop, the perturbation frequency must be selected such that it is slower than the slowest plant dynamics. Therefore plans appears as a static system to ESC. The cut-off frequencies of high-pass and low-pass filters must be designed in coordination with the perturbation frequency ω; $\omega_h<\omega$ and $\omega_l<\omega$. However, these filters should have sufficient bandwidth (higher cut-off

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