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Composite disturbance rejection control based on generalized extended state observer

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A B S T R A C T

Traditional extended state observer (ESO) design method does not focus on analysis of system reconstruction strategy. The prior information of the controlled system cannot be used for ESO implementation to improve the control accuracy. In this paper, composite disturbance rejection control strategy is proposed based on generalized ESO. First, the disturbance rejection performance of traditional ESO is analyzed to show the essence of the reconstruction strategy. Then, the system is reconstructed based on the equivalent disturbance model. The generalized ESO is proposed based on the reconstructed model, while convergence of the proposed ESO is analyzed along with the outer loop feedback controller. Simulation results on a second order mechanical system show that the proposed generalized ESO can deal with the external disturbance with known model successfully. Experiment of attitude tracking task on an aircraft is also carried out to show the effectiveness of the proposed method.

1. Introduction

System uncertainties, such as parameters perturbation, unmodeled dynamics, external disturbances, and sensor noise, will have great influence on the performance of a control system, even cause instability. It is not an easy work to design a controller which guarantees both disturbance rejection and tracking performance simultaneously with complicated uncertainties. Thus, composite disturbance rejection methodology with both outer loop controller and inner loop observer has been widely concerned [1]. For the composite disturbance rejection control system, the control accuracy is largely determined by the estimation accuracy of inner loop observer. There have been several observer design approaches investigated so far, such as disturbance observer [2], extended state observer (ESO) [3], unknown input observer [4], perturbation observer [5], equivalent input observer [6], sliding mode observer [7], and fuzzy observer [8–10]. Among these works, ESO needs the least prior information, even if the relative order of the plant is unknown [11]. On the other hand, comparing with the output observers, ESO can estimate not only the equivalent disturbance, but also the internal system states. Thus, state feedback controller can be designed for ESO based control system. According to these advantages, ESO based control, also known as active disturbance rejection control (ADRC), has been widely explored in recent years. It is reported that ESO has been employed in many successful applications, such as uncalibrated visual servoing [12], flight control [13], vibration control [14], power electronics [15], motor control [16]. In addition, various theoretical analyses have been explored based on ESO, such as Lyapunov stability analysis [17], parameter tuning strategy [18], and generalized ESO design for system of mismatched uncertainties [19].

As for the ESO based control structure, observation performance will largely determine the control performance of closed-loop system. Thus, various results on convergence analysis have appeared. For some researches, it is assumed that the change rate of uncertainty is bounded [13,15,19,20]. Then the estimation error of the ESO remains bounded, and its upper bound decreases monotonously when increasing the bandwidth of the observer. By introducing assumption on the system uncertainty, Lyapunov stability analysis of both nonlinear ESO is proposed in [17]. However, the above analysis is only proposed for traditional ESO design.

Despite the theoretical tools for convergence analysis, the specific disturbance rejection performance is rarely investigated, especially for different kinds of time-varying disturbances. It can be obtained in [21] that typical ESO offers asymptotic convergence of estimation for constant disturbance. However, time-varying disturbance, which is widely existing in practice, cannot be estimated by traditional ESO thoroughly [21]. Thus, it is important to explore the observer design methodology against time-varying disturbance for better disturbance rejection performance.

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In [22], the generalized ESO with high order is investigated, showing that it improves in the tracking of fast time-varying sinusoidal disturbances. From the results, it can be seen that the high order ESO can improve the estimation accuracy of sinusoidal external disturbances more or less. However, there still exists a periodic estimation error, which will in turn decrease the control accuracy of the closed-loop system [22]. According to internal model principle, the observer cannot reject the disturbance exactly unless the disturbance dynamics is embedded into the observer. In this paper, comparing with the high order ESO [22], the internal model principle is applied for generalized ESO implementation.

This paper devotes to increase the estimating accuracy of ESO against time-varying external disturbances. The definition of extended state for ESO is essentially the reconstruction for the controlled object. However, the existing researches only focus on the performance of ESO with different orders, while the prior information of system uncertainties cannot be further used for ESO implementation. Thus, we first analyze the reconstruction strategy of traditional ESO, and its limitation in dealing with time-varying disturbances is pointed out. To solve this problem, the system is reconstructed based on the model of the system uncertainties, and then the generalized ESO is proposed. At last, stability of the closed-loop system is analyzed along with the outer loop controller.

The rest of this paper is organized as follows. In Section 2, disturbance rejection performance of the traditional ESO is analyzed to show its limitations when dealing with time-varying disturbance. In Section 3, the controlled object is reconstructed by taking the disturbance model into account. Thus, a generalized ESO strategy for composite disturbance rejection is proposed. In Sections 4 and 5, both simulation and experiment are carried out to verify the effectiveness of the proposed strategy, followed by Conclusions in Section 6.

2. Problem statement

2.1. Traditional ESO

Consider the following uncertain single-input single-output (SISO) system, depicted by [17]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ x_n = f(t, x_1, x_2, \ldots, x_n) + w(t) + u(t) \\ y(t) = x_1, \end{cases} \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are the system states, u and y are the control input and output, respectively. w is the external disturbance, $f(\cdot)$ is the equivalent disturbance caused by both internal uncertainty and external disturbance.

In typical ESO an augmented variable $x_{n+1} \triangleq f(t, x, x_1, \ldots, x_n) + w(t)$ is introduced, such that the system can be reconstructed as:

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu + Eh \\ y = C\boldsymbol{x}, \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{(n+1)\times(n+1)}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix}_{(n+1)\times 1}, \quad E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(n+1)\times 1},$$

$$C = [1\ 0 \cdots 0\ 0]_{1\times(n+1)},$$

$$h = \frac{\mathrm{d}f(t, x_1, x_2, \ldots, x_n(t))}{\mathrm{d}t} + \frac{\mathrm{d}w(t)}{\mathrm{d}t}.$$

Then the linear ESO can be designed as follows:

$$\begin{cases} \dot{\boldsymbol{z}} = A\boldsymbol{z} + Bu + L(y - \hat{y}) \\ \hat{y} = C\boldsymbol{z}, \end{cases} \tag{3}$$

where $\boldsymbol{z} \in \mathbb{R}^{n+1}$, $L = [l_1\ \ l_2\ \ \cdots\ \ l_{n+1}]^{\mathrm{T}}$ such that all the roots of $s^{n+1} + l_1 s^n + \cdots + l_n s + l_{n+1} = 0$ are located at the right half s-plane. The selection of the observer gain is investigated in [18] based on bandwidth theory, which has been widely concerned in practice.

2.2. Disturbance rejection performance analysis

By introducing the Laplace transformation upon Eqs. (2) and (3), we get

$$z_{n+1} = H(s)x_{n+1}, \quad H(s) = \frac{l_{n+1}}{s^{n+1} + l_1 s^n + \cdots + l_n s + l_{n+1}}. \tag{4}$$

where $H(s)$ is an equivalent filter in the disturbance rejection structure. It is clearly that the control accuracy of the closed-loop system relies on the accuracy of the estimated disturbance z_{n+1}.

Assume that $\lim_{t\to\infty} x_i(t) = 0, i = 1, \ldots, n$. Without loss of generality, it is assumed that

$$\lim_{t\to\infty} f(t, x_1, x_2, \ldots, x_n) = 0.$$

Then we get

$$\begin{aligned} \lim_{t\to\infty} \tilde{d}(t) &= \lim_{t\to\infty}(x_{n+1} - z_{n+1}) = \lim_{s\to 0} s(1 - H(s))w(s) \\ &= \lim_{s\to 0} \frac{s^n + l_1 s^{n-1} + \cdots + l_n}{s^{n+1} + l_1 s^n + \cdots + l_n s + l_{n+1}} s^2 w(s). \end{aligned} \tag{5}$$

From Eq. (5), it can be found that if $w(t)$ is a constant disturbance, then $w(s) = \frac{1}{s}$ and $\lim_{t\to\infty} \tilde{d}(t) = 0$. Otherwise, there exists an estimation error, or the system even diverges. The estimation error can also be analyzed in the time domain.

By defining the estimation error of ESO as $\boldsymbol{e} = \boldsymbol{z} - \boldsymbol{x}$, the dynamic equation of $\boldsymbol{e}$ is described as:

$$\dot{\boldsymbol{e}} = (A - LC)\boldsymbol{e} - Eh. \tag{6}$$

When the system is in the steady state, the system states have converged to the equilibrium point. Then, the component of perturbation h that relies on the system states can be regarded as a constant. Thus, the component that relies on the external disturbance w will have a persistent influence on the control system. Three situations with different external disturbances can be analyzed as follows:

1. When the disturbance satisfies $\frac{\mathrm{d}w(t)}{\mathrm{d}t} = 0$, Eq. (6) turns to $\dot{\boldsymbol{e}} = (A - LC)\boldsymbol{e}$. Since $(A - LC)$ is a Hurwitz matrix according to the definition of A, L and C, the estimation error can converge to 0 exponentially.

2. When the disturbance satisfies $\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \text{Constant}$, the equilibrium point of Eq. (6) is $\boldsymbol{e}_0 = (A - LC)^{-1} E\frac{\mathrm{d}w(t)}{\mathrm{d}t}$. At this time, there exists a constant error of the estimation of ESO. The observation error is proportional to $|\frac{\mathrm{d}w(t)}{\mathrm{d}t}|$, is inversely proportional to the bandwidth of ESO.

3. For the other kinds of disturbances, $\frac{\mathrm{d}w(t)}{\mathrm{d}t}$ is a time-varying signal, the observation error $\boldsymbol{e}$ can only converge in a compact set, whose bound relies on the upper bound of $|\frac{\mathrm{d}w(t)}{\mathrm{d}t}|$ and the bandwidth of ESO.

According to the above analysis, the time-varying disturbance will have a significant influence on the control performance. In some cases, the model of the equivalent disturbance can be obtained, it is necessary for an observer to estimate the component with known model thoroughly to increase the control accuracy.