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Research Article

Study of the fractional order proportional integral controller for the permanent magnet synchronous motor based on the differential evolution algorithm

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ABSTRACT

A tuning method of the fractional order proportional integral speed controller for a permanent magnet synchronous motor is proposed in this paper. Taking the combination of the integral of time and absolute error and the phase margin as the optimization index, the robustness specification as the constraint condition, the differential evolution algorithm is applied to search the optimal controller parameters. The dynamic response performance and robustness of the obtained optimal controller are verified by motor speed-tracking experiments on the motor speed control platform. Experimental results show that the proposed tuning method can enable the obtained control system to achieve both the optimal dynamic response performance and the robustness to gain variations.

1. Introduction

In recent years, fractional calculus has been widely used in system modeling and control area [1–5]. Fractional calculus extends the descriptive ability of the integer calculus. Therefore, the characteristics of real-world systems can be described more precisely using the fractional order mathematical models [6,7]. PID control has been the most widely used and developed control method in the industrial control area [8]. With the introduction of the fractional order controlled models and the rising requirements of the control performance, the traditional PID control needs to be improved. The fractional order $PI^{\lambda}D^{\mu}$ controller, proposed by Podlubny in 1999 [9], has the potential to achieve better control performance because the differential order and integral order are introduced as adjustable controller parameters, increasing the flexibility of the controller. At present, the tuning methods of the fractional order $PI^{\lambda}D^{\mu}$ controller mainly include: the gain crossover frequency and phase margin method [10–14], the dominant pole method [15] and other optimization methods [16,17].

The gain crossover frequency and phase margin method calculates the controller parameters using the robustness specification, based on the given gain crossover frequency and phase margin [10-14]. The obtained control system has the specified crossover frequency and phase margin, as well as the robustness to gain variations. However, by now there are no generally accepted criteria for the selection of the gain crossover frequency and phase margin. Thus, the gain crossover frequency and phase margin method cannot ensure the control system to achieve the optimal dynamic response performance.

The dominant pole method calculates a pair of dominant poles for the system based on the requirement of the dynamic response performance. Then the controller parameters are obtained by using some optimization algorithms [15]. Other optimization methods search for the optimal controller parameters by optimizing a specified objective function [16,17]. The dominant pole method and other optimization methods can enable the obtained control system to achieve the optimal dynamic response performance. However, without the consideration of the characteristic in frequency domain, these methods cannot guarantee the control system to be robust to gain variations.

In order to obtain an optimal controller that enables the control system to achieve both the optimal dynamic response performance and the robustness to gain variations, a tuning method of the fractional order PI^{λ} speed controller for the permanent magnet synchronous motor (PMSM) speed control system is proposed in this paper. Taking the combination of the integral of time and absolute error (ITAE) and the phase margin as the optimization

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index, the robustness specification as the constraint condition, the differential evolution (DE) algorithm is applied to search the optimal crossover frequency, the phase margin and the controller parameters that enable the optimization index to reach the optimum.

The rest of the paper is arranged as follows: the fractional order controller tuning method is proposed in Section 2; the procedure of the tuning algorithm is discussed in Section 3; in Section 4, motor speed-tracking experiments are performed to verify the dynamic response performance and robustness of the obtained control system; the conclusion is given in Section 5.

2. Fractional order PI^{λ} controller tuning method

The fractional order PI^{λ} controller is described by (1),

$$C(s) = K_p\left(1+\frac{K_i}{s^{\lambda}}\right), \tag{1}$$

where K_p is the proportional gain, K_i is the integral gain and λ is the fractional order, the range of the fractional order is $0<\lambda<2$.

The controlled model in this paper is based on the fractional order open-loop transfer function of the PMSM speed control system [18]. The controlled model has the following form:

$$G(s) = \frac{c}{s^{\alpha}+as^{\beta}+b}, \tag{2}$$

where α and β are the fractional orders.

The robustness specification is described by (3), where ω_c represents the gain crossover frequency. The derivative of phase is zero, i.e., the phase Bode plot is flat at the gain crossover frequency. It means that the system is robust to tiny gain changes and the overshoots of the response are almost the same [11].

$$\left[\frac{d[\mathrm{Arg}(C(j\omega)G(j\omega))]}{d\omega}\right]_{\omega=\omega_c} = 0. \tag{3}$$

The robustness specification is introduced as the constraint condition of the optimization algorithm in this paper. From (3), we obtain,

$$\left[\frac{d[\mathrm{Arg}(C(j\omega)G(j\omega))]}{d\omega}\right]_{\omega=\omega_c} = \frac{d}{d\omega}[\mathrm{Arg}(G(j\omega))]_{\omega=\omega_c} + \frac{d}{d\omega}[\mathrm{Arg}(C(j\omega))]_{\omega=\omega_c}. \tag{4}$$

Specifically,

$$\frac{d}{d\omega}[\mathrm{Arg}(C(j\omega))]_{\omega=\omega_c} = \frac{-\lambda K_i \sin\left(\frac{\pi}{2}\lambda\right)\omega_c^{-\lambda-1}}{\omega_c^{-2\lambda}K_i^2+2\omega_c^{-\lambda}K_i\cos\left(\frac{\pi}{2}\lambda\right)+1}. \tag{5}$$

Introducing $M(j\omega_c) = \frac{d}{d\omega}[\mathrm{Arg}(G(j\omega))]_{\omega=\omega_c}$ yields,

$$\frac{-\lambda K_i \sin\left(\frac{\pi}{2}\lambda\right)\omega_c^{-\lambda-1}}{\omega_c^{-2\lambda}K_i^2+2\omega_c^{-\lambda}K_i\cos\left(\frac{\pi}{2}\lambda\right)+1} + M(j\omega_c) = 0. \tag{6}$$

Introducing $N(j\omega_c) = 2M(j\omega_c)\omega_c^{-\lambda}\cos(\frac{\pi}{2}\lambda) - \lambda\sin(\frac{\pi}{2}\lambda)\omega_c^{-\lambda-1}$, yields,

$$\omega_c^{-2\lambda}K_i^2M(j\omega_c)+K_iN(j\omega_c)+M(j\omega_c) = 0. \tag{7}$$

Solving the equation, yields,

$$K_i = \frac{-N(j\omega_c)\pm\sqrt{N(j\omega_c)^2-4M(j\omega_c)^2\omega_c^{-2\lambda}}}{2\omega_c^{-2\lambda}M(j\omega_c)}. \tag{8}$$

Once the values of the crossover frequency ω_c and fractional order λ are confirmed, K_i can be calculated using (8), and K_p can be calculated according to the definition of the gain crossover frequency. Therefore, the tuning of the fractional order controller is converted into the search of the optimal gain crossover frequency and fractional order.

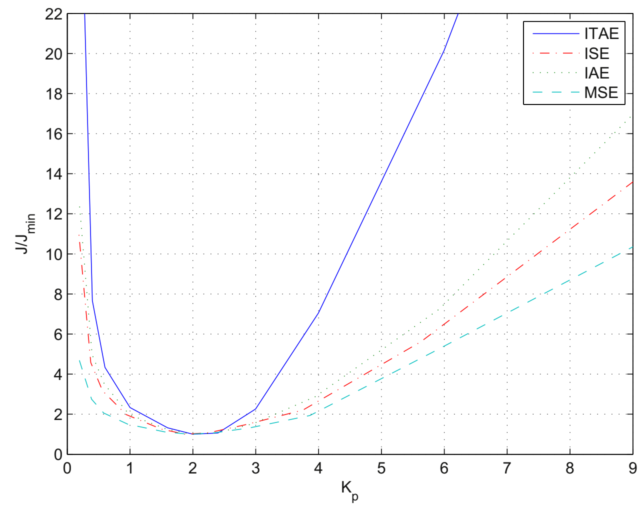

Fig. 1. The characteristics of different optimization index. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

The ITAE criterion is selected to be the optimization index of the algorithm because of the good selectivity, namely, the change of system parameters can obviously affect the ITAE value [19], which is a significant advantage for parameter optimization.The ITAE index is described by

$$J_{ITAE} = \int_0^{\infty} t\,|\,e(t)\,|\,dt, \tag{9}$$

where $e(t)$ represents the deviation between the specified value and the actual value.

In order to verify the selectivity, four PI controllers are designed for a plant model, using the ITAE, ISE (integral of squared error), IAE (integral of absolute error) and MSE (mean squared error) as the optimization index. Four groups of simulations are performed using these four controllers to control the plant model respectively. In each group of simulations, the gain of the corresponding controller is forced to deviate from the optimal value. The simulation outputs are measured and then the index J corresponding to different gains are calculated. For each optimization index, the ratio between the index values corresponding to different controller gains and the optimal value, J/J_{min} is calculated and plotted in Fig. 1, where the blue curve represents the ratio of ITAE when K_p deviates from the optimal value, the red curve represents that of ISE, the green curve represents that of IAE and the cyan curve represents that of MSE.

Fig. 1 shows that when the controller gains deviate from their optimal values, the variation of ITAE is much larger than those of other optimization index. Therefore, the ITAE index is the most sensitive to the deviation of parameters.

In order to obtain the optimal dynamic response performance and guarantee the system stability, the combination of the ITAE and phase margin is selected to be the optimization index of the algorithm. The optimization index of the algorithm is described by

$$J = \frac{w_1}{J_{ITAE}} + w_2\varphi_m, \tag{10}$$

where φ_m represents the phase margin, w_1 and w_2 are the weights of the ITAE and phase margin, respectively.

The DE algorithm is applied to search the optimal crossover frequency ω_c and fractional order λ. The fractional order controller, obtained from each pair of (ω_c,λ), is used for step response simulation. The optimization index J of each (ω_c,λ) pair is then