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# Research Article

## Robust absolute stability criteria for uncertain Lurie interval time-varying delay systems of neutral type

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#### ARTICLE INFO

### ABSTRACT

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Keywords: Neutral system Lurie dynamical system Interval time-varying delay Linear matrix inequality (LMI) Integral inequality approach (IIA) This study investigates the delay-dependent robust absolute stability analysis for uncertain Lurie systems with interval time-varying delays of neutral type. First, we divide the whole delay interval into two segmentations with an unequal width and checking the variation of the Lyapunov–Krasovskii functional (LKF) for each subinterval of delay, much less conservative delay-dependent absolute and robust stability criteria are derived. Second, a new delay-dependent robust stability condition for uncertain Lurie neutral systems with interval time-varying delays, which expressed in terms of quadratic forms of linear matrix inequalities (LMIs), and has been derived by constructing the LKF from the delayed decomposition approach (DDA) and integral inequality approach (IIA). Finally, three numerical examples are given to show the effectiveness of the proposed stability criteria.

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#### 1. Introduction

It is well-known that time delay is often an important source of instability, which is encountered in various engineering systems, such as nuclear reactors, chemical engineering systems, biological systems and population dynamics models. Hence, the stability analysis for time delay systems has great importance both in theory and in practice [9,10,22]. Neutral time delay systems are frequently encountered in many practical situations such as chemical reactors, water pipes, population ecology and so on [4,6,7,9,13,15–17].

On the other hand, nonlinear perturbations and parameter uncertainties may cause instability and poor performance of many practical systems. One of the important classes of nonlinear systems is the Lurie system whose nonlinear element satisfies certain sector constraints. Since the notion of absolute stability was introduced by Lurie [18], stability analysis for the Lurie systems has been extensively studied in [2,3,5,6–12,17–33]. Many researchers have investigated the absolute stability of Lurie control systems with time-delays and derived some stability criteria [20,21], but most of the existing criteria are delay-independent which do not include the information on time delay. Generally, abandonment of information on the delay causes conservativeness of the criteria especially when the delay is comparatively small.

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Delay-dependent sufficient conditions for the absolute stability of general neutral type Lurie direct control systems have been given in [9,23,24,27,32]. Gao et al. [9] investigated robust stability criteria in terms of linear matrix inequalities (LMIs) for uncertain neutral systems with time-varying delays and sector-bounded nonlinearity by employing the free-weighting matrix approach. Jensen integral inequality was taken in [23,24]. By eliminating nonlinearity and removing free-weighting matrices, novel sufficient conditions for the robustly asymptotic stability have been obtained [27]. By constructing a suitable augmented Lyapunov functional and using an integral inequality and free-weighting matrix techniques, Yin et al. [32] derived delay-dependent robust stability criteria for uncertain neutral systems with interval timevarying delays and sector-bounded nonlinearity. In practice, timevarying interval delay is often encountered: range of delay varies in an interval for which the lower bound is not restricted to 0. In this case, the stability criteria in [6,9,16,17,24] are conservative, since they do not take into account information on the lower bound of delay. In [7,23,27,32], the delay lower bound is not restricted to zero but it is a small positive number thus giving the measure of delay range. Up to now, to the best of the authors' knowledge, few results have been reported in literature for neutral type Lurie uncertain systems with time-varying interval delay. Still, estimating upper bound of Lyapunov functional derivative for time-varying systems with different time delay range leaves room for investigation. These facts motivate the present studies.







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However, there still exists room for further improvement because some important terms, which may lead to conservatism to some extent, where ignored during the estimation of its derivative in [8,9,11,22,26,29]. However, tighter estimation on its time derivative terms challenging and important, which is the motivation for this paper.

Fortunately, to the best of our knowledge, the problem of delay-dependent robust absolute stability criteria for neutral type Lurie uncertain systems with interval time-varying delays has not been fully studied in the literature and still remains open. Motivated by the above mentioned analysis, new delay-dependent criterions for the neutral type Lurie uncertain systems with timevarying delays are established. Contributions of this paper can be summarized as follows aspects. First, we divide the whole delay interval into two segmentations with an unequal width and checking the variation of the LKF for each subinterval of delay, much less conservative delay-dependent absolute and robust stability criteria are derived. Second, a new delay-dependent robust stability condition for uncertain Lurie neutral systems with interval time-varying delays, which expressed in terms of guadratic forms of linear matrix inequalities (LMIs), and has been derived by constructing the Lyapunov-Krasovskii functional (LKF) from the delayed decomposition approach (DDA) and integral inequality approach (IIA). Third, all the conditions are presented in terms of linear matrix inequalities (LMIs) can be easily calculated by using Matlab LMI control toolbox. Finally, three numerical examples are given to show the effectiveness of the proposed stability criteria.

#### 1.1. Notations

Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively;  $R^{n \times n}$  denotes an *n*-dimensional Euclidean space;  $R^{m \times n}$  is the set of all  $m \times n$  real matrices; P > 0 means that matrix *P* is symmetric positive definite; for real symmetric matrices *X* and *Y*, the notation  $X \ge Y$  (respectively, X > Y) means that the matrix X - Y is positive semi-definite (respectively, positive definite); *I* is an appropriately dimensional identity matrix;  $X_{ij}$  denotes the element in row *i* and column *j* of matrix *X*; the notation \*always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

#### 2. Problem formulation

Consider the following uncertain neutral type of Lurie system with interval time-varying delays and sector-bounded nonlinearity:

$$\mathbf{x}(t) - (C + \Delta C(t))\mathbf{x}(t - \tau(t)) = \downarrow (A + \Delta A(t))\mathbf{x}(t) + (A_1 + \Delta A_1(t))\mathbf{x}(t - h(t))$$

$$\downarrow + (B + \Delta B(t))f(\sigma(t)), \tag{1a}$$

$$\sigma(t) = D^T x(t), t \ge 0, \tag{1b}$$

$$x(s) = \phi(s), s \in [-\max\{h_2, \tau\}, 0],$$
(1c)

where x(t) is the *n*-dimensional state vector,  $\sigma(t)$  is the *m*-dimensional output vector,  $f(\sigma(t))$  is an  $R^m$ -valued nonlinear function in the feedback path,  $\phi(s)$  is an  $R^n$ -valued continuous initial function specified on  $[-\max\{h_2, \tau\}, 0]$  with known positive scalars  $h_2$  and  $\tau$ ,  $\Delta A(t)$ ,  $\Delta A_1(t)$ ,  $\Delta B(t)$  and  $\Delta C(t)$  are time-varying uncertainties, h(t) and  $\tau(t)$  represent time-varying delays, and A,  $A_1, B, C$ , and D are known real constant matrices with appropriate dimensions.

Throughout this paper, we make the following assumptions.

**Assumption 1.** We assume that  $\rho(C) < 1$ .

**Assumption 2.** The time-varying delays 
$$h(t)$$
 and  $\tau(t)$  satisfy

$$< h_1 \le h(t) \le h_2, h(t) \le h_d, 0 \le \tau(t) \le \tau, \dot{\tau}(t) \le \tau_d < 1,$$
 (2)

where  $h_1, h_2, h_d, \tau$  and  $\tau_d$  are known scalars.

**Assumption 3.** [22,23,26,31]. The nonlinear functional  $f(\sigma(t)) \in \mathbb{R}^m$  in the feedback path is given by form

$$f(\sigma(t)) = \begin{bmatrix} f_1(\sigma_1(t)) & f_2(\sigma_2(t)) & \dots & f_m(\sigma_m(t)) \end{bmatrix}^T,$$
(3)

And

$$\sigma(t) = \begin{bmatrix} \sigma_1(t) & \sigma_2(t) & \dots & \sigma_m(t) \end{bmatrix}^T$$
$$= \begin{bmatrix} d_1 x(t) & d_2 x(t) & \dots & d_m x(t) \end{bmatrix}^T,$$
(4)

Wherein, each term  $f_i(\sigma_i(t))(i = 1, 2, ..., m)$  satisfies the infinite sector condition

$$f_i(\sigma_i(t)) \in K_i[0,\infty] = \{f_i(\sigma_i(t)) | f_i(0) = 0, \sigma_i(t) f_i(\sigma_i(t)) > 0\},$$
(5)

or the finite sector condition

$$f_i(\sigma_i(t)) \in K_i[0, k_i] = \{f_i(\sigma_i(t)) | f_i(0) = 0, 0 < \sigma_i(t) f_i(\sigma_i(t)) \le k_i \sigma_i^2(t), \sigma_i \neq 0\},$$
(6)

with known positive scalars  $k_i$  (i = 1, 2, ..., m).

**Assumption 4.** The time-varying uncertainties  $\Delta A(t)$ ,  $\Delta A_1(t)$ ,  $\Delta B(t)$  and  $\Delta C(t)$  are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta A_1(t) & \Delta B(t) & \Delta C(t) \end{bmatrix} = MF(t) \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}, \quad (7)$$

where  $M, N_1, N_2, N_3$  and  $N_4$  are constant matrices with appropriate dimensions, and F(t) is an unknown, real, and possibly timevarying matrix with Lebesgue-measurable elements satisfying

$$F^{T}(t)F(t) \le I, \qquad \forall t \ge 0.$$
(8)

The purpose of this paper is to investigate several robustly asymptotic absolute stability criteria for the system (1) under these assumptions mentioned above. When the time delay is unknown, how long can time delay be tolerated to keep the system stable? To do this, three fundamental lemmas are reviewed.

Lemma 1. [13–16]. For any positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0,$$
 (9a)

the following integral inequality holds

$$-\int_{t-h(t)}^{t} \dot{x}^{T}(s) X_{33} \dot{x}(s) ds$$

$$\leq \int_{t-h(t)}^{t} \begin{bmatrix} x^{T}(t) & x^{T}(t-h(t)) & \dot{x}^{T}(s) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^{T} & X_{22} & X_{23} \\ X_{13}^{T} & X_{23}^{T} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds.$$
(9b)

**Lemma 2.** [1]. The following matrix inequality

$$\begin{vmatrix} Q(x) & S(x) \\ S^{T}(x) & R(x) \end{vmatrix} < 0,$$
(10a)

where  $Q(x) = Q^{T}(x)$ ,  $R(x) = R^{T}(x)$  and S(x) depend on affine on x, is equivalent to

$$R(x) < 0, \tag{10b}$$

$$Q(x) < 0, \tag{10c}$$

and

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