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On robust control of continuous-time systems with state-dependent uncertainties and its application to mechanical systems

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ABSTRACT

This paper revisits the problems of robust stability analysis and control of continuous-time systems with state-dependent uncertainties. First, a more general polytopic model describing systems with state-dependent uncertain parameters is proposed, and such a system model is more applicable in practice. A low conservative stability condition is obtained for the system by introducing the Lagrange multiplier term and adding some weight matrix variables. Then, based on our proposed idea, the output-feedback controllers will be designed in two cases: (1) the system matrices share the same polytopic parameters; (2) the system matrices do not share the same polytopic parameters. The controllers are designed in a model-dependent manner, which can provide more flexibilities in control synthesis. Besides, a decay rate can be set in advance to achieve better system performances. Finally, a numerical example together with a classic mechanical system is used to demonstrate the effectiveness and applicability of our theoretical findings.

1. Introduction

As a branch of control theory, robust control has been widely studied in the past decades [1–3] due to its numerous applications in many practical systems such as flight control systems [4], mechanical systems [5,6], networked control systems [7,8], and electronic circuits [9]. The goal of robust control theory is to provide a method to measure the performance changes and avoid big performance degradation in the presence of uncertainties or partial system faults [10,11].

Uncertainties are often encountered in various types of systems with parameter variations, un-model dynamics, and/or disturbance inputs, and some well developed techniques from robust control can deal with the control problems of systems subject to these uncertainties. Generally speaking, system uncertainties comprise disturbance signals [12,13] and dynamic perturbations [14,15]. The former include sensor noise, input disturbance, etc., and the latter are often called structure uncertainties reflecting the discrepancy between the mathematical model and the actual system.

System parametric variation, caused by environmental changes or torn-and-worn factors, is a form of structured uncertainty [16,17]. In practice, parametric uncertainties can be the values of the friction, stiffness or inertia coefficients, etc., in mechanical systems; resistor or inductance, etc., in electronic circuit systems; and the aerodynamical coefficients in flight control systems, to list a few. Considering the wide existences of parametric uncertainties, many researchers have devoted their attention to robust control of such uncertain systems. In modeling systems with parametric uncertainties, it is desirable to have models which closely match the reality, and are easy to analyze. The convex polytope-type model that describes the realistic parameter uncertainty has been recognized to be more general than the well-known interval and linear parameter uncertainty as well as multi-model structures [18]. Note that such a description has found its natural framework in the linear matrix inequality (LMI) formalism [19].

Compared with the μ/k_m theory and algebraic approaches based on Edge theorem or Kharitonov's theorem, the quadratic Lyapunov function approach has been verified to be an efficient and powerful tool in the LMI framework for robust analysis and synthesis of systems with polytopic uncertainties [20]. Here, it is worth pointing out that, for not only practical but also technical reasons, it is of importance to divide the uncertain parameters into the following two categories: constant uncertain parameters and time-varying uncertain parameters. Up to now, robust control of systems with both types of parametric uncertainties has been studied by quadratic Lyapunov function in the literature.

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To ensure the stability against arbitrary fast parameter variations, the common quadratic Lyapunov function (CQLF) composed of a single Lyapunov function is always used for testing stability over the whole uncertainty polytope. As a result, the CQLF approach is very conservative for constant or slowly time-varying uncertain parameters [21,19,22]. To reduce the conservatism in analyzing robust stability of polytopic systems with constant uncertain parameters, the parameter dependent Lyapunov function (PDLF) is introduced in both continuous-time and discrete-time contexts [23], which is quadratic on the system states and depends affinely on the uncertain parameters [24,25,22]. Similarly, the PDLF method can also give less conservative results for polytopic systems with slowly time-varying parameters provided that the bounds of the variation rate of the parameters are known. To get improved results in case of slowly time-varying uncertainties, some other types of Lyapunov functions such as piecewise Lyapunov function, polynomially parameter-dependent Lyapunov function, and biquadratic Lyapunov function [26,27], have been recently proposed in the literature. Meanwhile, it should be emphasized that, it is hard or impossible to estimate the bounds of variation of time-varying parameter in practice, which will reduce the applicability of the corresponding results.

On the other hand, in many practical systems, the values of time-varying uncertain parameters are, more precisely, state-dependent, see for example, spring stiffness and friction coefficient in mechanical systems, nonlinear resistor and tunnel diode in electronic circuits [28–30]. More recently, the robust control for systems with state-dependent uncertainties has been investigated in both nonlinear and linear cases [31]. In the linear case, traditional criteria by PDLF approach require the bounds of variation of uncertain parameter, whereas it is hard to get the ideal knowledge on time derivatives of state-dependent uncertain parameters, or the cost is probably high. Furthermore, for state-dependent uncertainties, additional requirements on the bounds will lead to unexpected restrictions in the state space. Therefore, for conservatism and applicability reasons, neither the CQLF nor the PDLF is effective for such systems. Then, a class of line integral Lyapunov function is introduced in [32] to get less conservative and applicable conditions.

However, the results in [32] still leave some rooms for improving, and give rise to the following issues: (1) the stability conditions are sufficient, and thus need to be further improved; (2) a local static output feedback controller is designed in [32], which cannot achieve better performances in some cases; (3) the proposed model in [32] requires that the uncertainties in system state matrix and control matrix are described by the same state-dependent parameter vector, which is obviously unrealistic in practice. Therefore, a more general type of modeling approach is necessary. (4) For control synthesis, only the stability of closed-loop is considered in [32], whereas the system performance is not taken into account.

In this paper, we revisit the problems of robust control for a class of linear uncertain systems with polytopic state-dependent uncertainties. The contributions of the paper exist in: a more general model describing uncertain system with polytopic state-dependent uncertainties is proposed, which is more applicable in practice; less conservative stability criteria for considered systems are obtained, and these stability conditions are also efficient for controller design. Moreover, the controller explored in this paper is model-dependent, and thus can provide more flexibilities in control synthesis. The remainder of the paper is organized as follows: Section 2 proposes a more general mathematical model for uncertain systems with state-dependent parameters, and reviews some necessary definitions and lemmas. In Section 3, some improved stability conditions are established, upon which, a set of model-dependent controllers are designed in Section 4. A numerical example and a classic mass–spring–damper system that has wide applications in mechanical systems are used in Section 5 to demonstrate the effectiveness and applicability of our theoretical findings. Finally, Section 6 concludes the paper.

Notations: In this paper, the notation $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\bar{x}$ stands for the assembled set by the entries of a given vector x; the positive natural numbers are denoted by $\mathbb{N}_+$; the empty set is represented by $\varnothing$; a $n \times n$ matrix $V \equiv [v_{ij}]_{n\times n}$; for any given matrix A, it is defined that $sym\{A\} = A + A^T$; in addition, the notation $P > 0$ means that P is a real symmetric and positive definite matrix.

2. Problem formulation and preliminaries

In this paper, we consider the following continuous-time systems with state-dependent uncertainties that exist in both system matrix $A(\sigma(x(t)))$ and control matrix $B(\epsilon(x(t)))$, having the following form:

$$\begin{cases} \dot{x}(t) = A(\sigma(x(t)))x(t) + B(\epsilon(x(t)))u(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $u(t) \in \mathbb{R}^{n_u}$ is the control input; $y(t) \in \mathbb{R}^{n_y}$ is the system output; $x(t) \in \mathbb{R}^n$ is the system state; $\sigma(x(t)) \in \mathbb{R}^m$ and $\epsilon(x(t)) \in \mathbb{R}^m$ are the uncertain parameter vectors with components $\sigma_i(x_\sigma^i(t))$ and $\epsilon_i(x_\epsilon^i(t))$ $i \in S = \{1, 2, \ldots, m\}$, representing the state-dependent unknown parametric perturbations; in addition, $x_\sigma^i(t)$ and $x_\epsilon^i(t)$ are vectors whose entries are elements of state $x(t)$; system (1)'s state matrix $A(\sigma(x(t)))$ and control matrix $B(\epsilon(x(t)))$ are described by the following convex polytopic sets:

$$\mathcal{A} = \left\{ A(\sigma(x(t)) \,|\, A(\sigma(x(t)) = \sum_{i=1}^{m} \sigma_i(x_\sigma^i(t))A_i, \sigma(x(t)) \in \mathcal{D}_\sigma \right\} \tag{2}$$

$$\mathcal{B} = \left\{ B(\epsilon(x(t))) \,|\, B(\epsilon(x(t))) = \sum_{i=1}^{m} \epsilon_i(x_\epsilon^i(t))B_i, \epsilon(x(t)) \in \mathcal{D}_\epsilon \right\} \tag{3}$$

where the components $\sigma_i(x_\sigma^i(t))$ and $\epsilon_i(x_\epsilon^i(t))$, $i \in S$, satisfy

$$\mathcal{D}_\sigma = \left\{ \sigma(x(t)) \in \mathbb{R}^m \left| \sum_{i=1}^{m} \sigma_i(x_\sigma^i(t)) = 1, \sigma_i(x_\sigma^i(t)) \geq 0, i \in S \right. \right\} \tag{4}$$

$$\mathcal{D}_\epsilon = \left\{ \epsilon(x(t)) \in \mathbb{R}^m \left| \sum_{i=1}^{m} \epsilon_i(x_\epsilon^i(t)) = 1, \epsilon_i(x_\epsilon^i(t)) \geq 0, i \in S \right. \right\} \tag{5}$$

Different from the system model in [32] requiring that $\sigma(x(t)) = \epsilon(x(t))$, system model (1) does not have such a restriction on the state-dependent uncertain parameter vectors $\sigma(x(t))$ and $\epsilon(x(t))$, which is obviously more applicable in practice. An practical example in the simulation part will demonstrate the advantage of system model (1).

Remark 1. For system (1), if the uncertainties in state matrix and control matrix are described by the same state-dependent parameter vector (i.e., $\sigma_i(x_\sigma^i(t)) = \epsilon_i(x_\epsilon^i(t))$), or the control matrix is constant, then system (1) will reduce to the system model in [32].

Before proceeding, the following useful lemma is first recalled, which will be used to develop our main results.

Lemma 1 (*LaSalle [33]*). *If the time derivative of the Lyapunov function candidate $V(x(t))$ of the autonomous system $\dot{x}(t) = f(x(t))$ is globally negative definite, i.e., $\dot{V}(x(t)) < 0, \forall x(t) \in \mathbb{R}^n \backslash 0^n$, then the*