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Nonlinear receding horizon guidance for spacecraft formation reconfiguration on libration point orbits using a symplectic numerical method

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ABSTRACT

This paper studies a nonlinear receding horizon control guidance strategy for spacecraft formation reconfiguration on libration orbits in the Sun–Earth system. For comparison, a linear quadratic soft terminal control strategy is also considered for the same reconfiguration missions. A novel symplectic iterative numerical algorithm is proposed to obtain the optimal solution for the nonlinear receding horizon control strategy at each update instant. With the aid of the quasilinearization method, a high-efficiency structure-preserving symplectic method is introduced in the iterations, and the optimal control problem is replaced successfully by a series of sparse symmetrical linear equations. Several typical spacecraft formation reconfiguration missions including resizing, rotating and slewing reconfigurations and their combinations are numerically simulated to show the effectiveness of the nonlinear receding horizon guidance strategy based on the proposed symplectic algorithm. Through these simulations, the nonlinear receding horizon control strategy is shown to have obvious advantages in convergence and parameter sensitivity compared with a linear quadratic soft terminal control strategy. Monte Carlo stochastic simulations are used to test the robustness of the nonlinear receding horizon control guidance in dealing with measurement and execution errors.

1. Introduction

Spacecraft formation flying on libration point orbits is essential to meet the increasing demand for deep space exploration [1–3]. However, deep space spacecraft formation flying based on the circular restricted three-body problem (CRTBP) dynamical model for a deep space mission is very different from spacecraft formation flying based on the Kepler dynamical model for near Earth missions [4–7]. The nature of the dynamical model of CRTBP does not provide an analytical solution, so the analysis of spacecraft formation flying is still strongly dependent on numerical methods. Thus far, potential future applications that involve spacecraft formation flying on the libration points, have attracted great interest in formation control strategies and numerical methods.

Various strategies and approaches have been proposed for spacecraft formation reconfiguration on libration point orbits. Optimal nonlinear control and geometric control methods have been derived and compared to more traditional linear quadratic regulators as well as input state feedback linearization [8]. A finite element method for solving optimal control has been developed and applied to spacecraft formation reconfiguration by using linearized equations about a nonlinear nominal base orbit [9]. A recently developed technique based on generating functions has been employed for designing spacecraft formation reconfiguration in Hill three-body dynamics [10]. Efficient parameter optimization algorithms for collision-free energy sub-optimal path planning for formations of spacecraft flying in deep space are presented [11]. Based on the characterization of requirements and constraints, different algorithms for centralized optimal formation planning and coordination have been developed [12]. A reconfiguration guidance algorithm based on finite dimensional parameter optimization for spacecraft formation is presented [13]. The above-mentioned studies on spacecraft formation reconfiguration near libration points mainly focus on path planning, especially avoiding collisions, reducing energy or fuel costs, minimizing reconfiguration time and balancing energy. However, the collinear libration point orbits are inherently unstable. Furthermore, any perturbations in real environments will move rapidly the spacecraft off the collinear libration point orbits. Therefore, it is meaningful to

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investigate a suitable guidance strategy for compensating for these disturbances.

Guidance methods such as reference trajectory guidance [14–16] and predictive guidance [17,18] can be used to correct deviations and compensate for disturbances. The reference trajectory guidance method is used to track a reference trajectory and is implemented with an online closed-loop feedback control [19]. The predictive guidance method can handle large dispersions and accommodate severe off-nominal conditions. With the rapid growth of digital computers, receding horizon control (RHC) has become a powerful tool for spacecraft predictive guidance missions [20,21]. The essential characteristics of RHC is that the optimal control problem is solved over a shorter and moving horizon. The successful applications of RHC guidance are due to its reducing of sensitivity from disturbances and parameter variations [22]. When using RHC guidance, the optimal control problem is solved over a shorter moving horizon. However, the online computational burden associated with solving the moving optimal control problem is an impediment to practical real-time applications.

The development of efficient numerical algorithms for the RHC problem is an active area of research, and many numerical methods have been proposed. Based on Simpson-trapezoid approximations for the integral and Euler-type approximations for the derivatives, reference [23] transformed the linear RHC problem into a quadratic programming problem. The indirect Jacobi pseudospectral method [24] expanded the state and costate variables into polynomials with the values of the states and costates at different discretization points as the expansion coefficients, reducing linear RHC problems into systems of algebraic equations. The main drawbacks of pseudospectral methods for a large state-space model with a large discretization of unknown variables is that the linear equation obtained is large and dense with an asymmetrical coefficient matrix, which increases computer memory storage needs and reduces on-line implementation efficiency. An efficient sparse symplectic numerical approach for solving the linear RHC problem is proposed [25]. To solve the nonlinear RHC problem, a fast numerical algorithm based on the generalized minimum residual (GMRES) method combined with the continuation method has been proposed [26].

Motivated by the requirements of a guidance strategy for spacecraft formation reconfiguration and a high-performance numerical algorithm for solving the nonlinear RHC problem, an efficient symplectic numerical algorithm based on continuous finite low-thrust for solving nonlinear RHC guidance of spacecraft formation reconfiguration on a given libration point orbit in the Sun–Earth system will be investigated in this paper. The contributions of this paper include the following two points: (1) an efficient and easily implemented numerical algorithm that combines quasilinearization techniques and symplectic preserving has been developed to solve the nonlinear RHC guidance strategy; (2) the features and behaviors of the presented nonlinear RHC guidance strategy for spacecraft formation reconfiguration, including the convergence of the symplectic algorithm and its parameter sensitivity, have been studied in detail.

2. Mathematical modeling for spacecraft formation reconfiguration on libration point orbits

2.1. Circular restricted three-body problem

In this paper, spacecraft formation reconfiguration missions on libration point orbits are modeled using the equations of motion of the circular restricted three-body problem (CRTBP). The reference coordinate frame (O,X,Y,Z), which is centered on the barycenter, rotates at the same rate as the orbital motion of the two massive bodies. The X axis extends from the barycenter through Earth, the Z axis extends in the direction of the angular momentum of the system, and the Y axis completes the right-handed coordinate frame. The equations that describe the motion of the spacecraft without control can be written in the dimensionless form [27,28]

$$\begin{cases} \ddot{X}-2\dot{Y}=\frac{\partial U}{\partial X}, \\ \ddot{Y}+2\dot{X}=\frac{\partial U}{\partial Y}, \\ \ddot{Z}=\frac{\partial U}{\partial Z}, \end{cases} \tag{1}$$

where $U=\frac{1}{2}\left(X^2+Y^2\right)+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{1}{2}\mu(1-\mu)$.

The dot in Eq. (1) represents the time derivative in the rotating frame. μ is the mass-ratio parameter used to nondimensionalize the system. $r_1=\sqrt{(X+\mu)^2+Y^2+Z^2}$ and $r_2=\sqrt{(X-(1-\mu))^2+Y^2+Z^2}$ are equal to the distances from the spacecraft to the Sun and Earth, respectively.

2.2. Relative dynamical model

Because this paper mainly discusses the problem of spacecraft formation reconfiguration around the L_2 libration point of the Sun–Earth system, it would be convenient to transfer the reference frame from the barycenter of the Sun–Earth system to the L_2 point. The relationship between the (O,X,Y,Z) reference frame and the new (L_2,x,y,z) reference frame is as follows:

$$x=\frac{(X-1+\mu-\gamma)}{\gamma},\quad y=\frac{Y}{\gamma},\quad z=\frac{Z}{\gamma} \tag{2}$$

In Eq. (2), the distance between the Earth and L_2 is denoted by γ, and it is taken as the new unit of length for convenience. The motion equations of virtual spacecraft on libration point orbits without control can then be transferred from Eq. (1) to the following equation

$$\begin{cases} \ddot{x}_o-2\dot{y}_o-x_o=-\frac{(1-\mu)(x_o+1+1/\gamma)}{\gamma^3 d_{1o}^3}-\frac{\mu(x_o+1)}{\gamma^3 d_{2o}^3}+\frac{1-\mu+\gamma}{\gamma} \\ \ddot{y}_o+2\dot{x}_o-y_o=-\frac{(1-\mu)y_o}{\gamma^3 d_{1o}^3}-\frac{\mu y_o}{\gamma^3 d_{2o}^3} \\ \ddot{z}_o=-\frac{(1-\mu)z_o}{\gamma^3 d_{1o}^3}-\frac{\mu z_o}{\gamma^3 d_{2o}^3} \end{cases} \tag{3}$$

where x_o, y_o and z_o denote the position coordinates of the virtual spacecraft on libration point orbits in the (L_2,x,y,z) reference frame, $d_{1o}=\sqrt{(x_o+1+1/\gamma)^2+y_o^2+z_o^2}$, and $d_{2o}=\sqrt{(x_o+1)^2+y_o^2+z_o^2}$.

Meanwhile, in the new (L_2,x,y,z) reference frame, the dynamical equation of the ith real spacecraft under the active control can be written as follows:

$$\begin{cases} \ddot{x}_i-2\dot{y}_i-x_i=-\frac{(1-\mu)(x_i+1+1/\gamma)}{\gamma^3 d_{1i}^3}-\frac{\mu(x_i+1)}{\gamma^3 d_{2i}^3}+\frac{1-\mu+\gamma}{\gamma}+u_{ix} \\ \ddot{y}_i+2\dot{x}_i-y_i=-\frac{(1-\mu)y_i}{\gamma^3 d_{1i}^3}-\frac{\mu y_i}{\gamma^3 d_{2i}^3}+u_{iy} \\ \ddot{z}_i=-\frac{(1-\mu)z_i}{\gamma^3 d_{1i}^3}-\frac{\mu z_i}{\gamma^3 d_{2i}^3}+u_{iz} \end{cases} \tag{4}$$

where x_i, y_i and z_i denote the position coordinates of the ith real spacecraft in the (L_2,x,y,z) reference frame, u_{ix}, u_{iy} and u_{iz} are the control inputs of the ith real controlled spacecraft in the x, y and z directions, $d_{1i}=\sqrt{(x_i+1+1/\gamma)^2+y_i^2+z_i^2}$, and $d_{2i}=\sqrt{(x_i+1)^2+y_i^2+z_i^2}$.

By subtracting Eq. (3) from Eq. (4), the nonlinear relative dynamical equation of the ith real spacecraft under the active control with respective to the virtual spacecraft without control on