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Robust controller synthesis for high order unstable processes with time delay using mirror mapping technique

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ABSTRACT

In this note, a general scheme is proposed for the high order unstable delay process with one or more positive poles, using the mirror mapping technique. The Nyquist criteria is employed to establish a systematic methodology to tune the parameter. The stabilizing parameter region could guarantee the prespecified robustness specification. In the scheme, a control law is designed based on the all-pole Padé approximated model. The unstable process was first mapped into a minimum-phase system, and the actual control is obtained by the closed-loop gain shaping algorithm (CGSA). The advantages are that one has a concise design procedure and can achieve good performance such as disturbance rejection and robustness. Finally, three highly cited examples are used to illustrate the effectiveness of the proposed method.

1. Introduction

Open-loop unstable plant is a class of special processes existed in the process industry such as the stirred chemical reactor, distillation columns and bioreactors [1]. The time delay feature often degrade the closed-loop stability or even lead to invalidation of the conventional control scheme. The control problem is more difficult when the high order dynamics and the multiple unstable poles are involved in the feedback stabilizing system. Thus, the control of unstable processes with time delay attracts the control community's attention and many papers have been published [2–4].

In the past years, the classical smith predictor has been adopted to compensate the effect of time delay, and lead to an improved closed-loop performance over that of the conventional single feedback control [5,6]. However, the prediction scheme is limited for the integrating unstable process. It is very fragile to the modeling mismatch error and the instable dynamic. Motivated by the observation, some enhanced smith predictors have been developed [7,4,8]. In [7], a local feedback compensation is incorporated together with the forward output estimation to guarantee prediction convergence. This extents the stabilizing robustness to the large time delay and unstable dynamic. Furthermore, the digital version in [8] is developed using the linear quadratic method. Contrary to the smith predictor, the prominent superiority from the single feedback control structure is its concise form and convenience to implement in practical engineering. Lee and Wang investigated the stabilization of unstable delay processes with the simple P/PI/PID controller [9–11,1], and complete stabilizability conditions were established by use of the Nyquist stability criterion. Shamsuzzoha [12] proposed an internal model control based PID (IMC-PID) synthesis algorithm, which has a single tuning parameter to adjust the performance and the robustness. The conclusion is just equal to that of [13], where the IMC-PID parameters are determined using the Laurent series. There also exists the other significant schemes for unstable delay processes, such as the polynomial Diophantine method [14,15], the eigenvalue-loci technique [16] and the IMC synthetic method [17].

Note that the aforementioned works are about unstable delay system with single right-half-plane (RHP) pole and 1st/2nd order. Increment of the system order and number of RHP poles would complicate the controller design procedure and blame the closed-loop performance. However, only few control strategies involve the case of high order unstable delay plant with one or more RHP poles in the current literatures [18,19]. As to the multi-scale scheme [18], the control structure would become complicated along with the increment of system order. It is challenging for the algorithm to be

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implemented in engineering. A graphical method for determining the feasible robust PID controller is presented in [19].

In the authors' previous works, the stabilization of unstable processes has been addressed by the novel mirror mapping technique, including the regular unstable process [20,21], the pure unstable process [22,23], the integrating unstable delay process [24]. These existed results have been applied to the maglev train [25] (unstable process with dual poles, i.e. $(s+\omega_1)(s-\omega_1)$) and the static Beam riding missile [26,27] (unstable process with dual poles and dual zeros, i.e. $\left[(s+\lambda_1)(s-\lambda_1)\right]/[(s+\omega_1)(s-\omega_1)]$) with satisfactory performances. Those works have illustrated the capability and effectiveness of the corresponding algorithm.

This note is engaged in robust synthesis of the high order unstable delay process with one or more positive poles. In the scheme, the all-pole Padé approximation is incorporated to address the time delay dynamic. Using the CGSA, the controller is derived from the mirror mapping model. Further, the Nyquist criteria and a gain-phase margin tester are employed to establish the exact stabilizing parameter region for the closed-loop system. Compared with the existed results, the proposed scheme has the advantage of concise design procedure, and only one parameter needs to be tuned targetly to obtain the desired performance.

2. Problem description and preliminaries

This note concentrates on the stabilization problem of unstable delay processes (1) with arbitrary system order $(n+\vartheta)$. n denotes number of the stable poles and ϑ is number of the unstable RHP poles. The typical single feedback structure is deliberately employed for its convenient realizability in industrial and chemical practice, see Fig. 1(a). Fig. 1(b) presents the control diagram with a gain-phase margin tester, which is useful for the robustness analysis and will be detailed in Section 3.3. In Fig. 1, $\overline{G}(s), \overline{C}(s)$ are the plant and the corresponding controller. The related variables r, e, u, y, d_1 and d_2 denote the reference input, stabilizing error, control input, system output, the load and output disturbances, respectively.

$$G(s)=\frac{\sum_{k=0}^{m} a_k s^k}{\sum_{j=0}^{n} b_j s^j \prod_{i=1}^{\vartheta}(\tau_i s-1)} e^{-\tau_d s} = \frac{a_m s^m + a_{m-1}s^{m-1}+\cdots+a_0}{(b_n s^n + b_{n-1}s^{n-1}+\cdots+b_0)(\tau_1 s-1)(\tau_2 s-1)\cdots(\tau_\vartheta s-1)} e^{-\tau_d s} \tag{1}$$

where $a_k \in R, b_j \in R$ are the real coefficients, which can guarantee that the corresponding component in $G(s)$ is stable. $a_m \neq 0, b_n \neq 0, n+\vartheta \geq m, \tau_i > 0$. $1/\tau_i, i=1,2,\ldots,\vartheta$ are the open RHP poles. When selecting $m=1, \vartheta=1, \tau_\vartheta=1$, the process (1) could be a particular version of the plant in [1], i.e., $G(s)=\frac{\alpha s+1}{(s-1)\prod_{j=1}^{n}(\beta_j s+1)} e^{-\tau_d s}$. Hence, Eq. (1) describes a class of high order unstable processes with more general form.

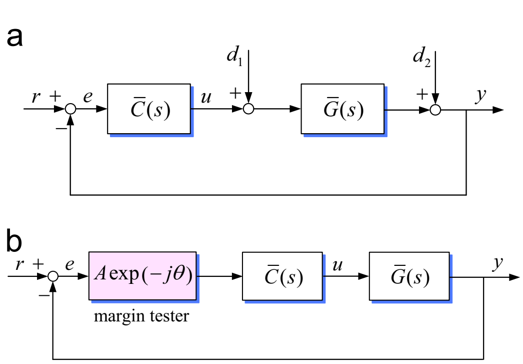

Fig. 1. Framework for control design and robust analysis. (a) The typical single feedback control system. (b) The control structure with a gain-phase margin tester.

In order to avoid generating the unwanted positive poles or zeros, the all-pole Padé approximation [28] is employed to transform the time delay component into the finite dimensional term. For instance, the 1st order Padé approximation is applied to the process (1) in the following analysis and synthesis, i.e. $e^{-\tau_d s} \approx 1/(\tau_d s+1)$. The control design is derived from the corresponding approximated model $G_A(s)$. It is obviously to note that, the perturbation error between the original unstable delay process and the approximated model decreases along with employing the higher order Padé approximation. As such, the time delay defect may be compensated effectively by the related control law, whereas with a more complicated form. Therefore, the 1st order all-pole Padé approximation is selected and incorporated in the following analysis:

$$G_A(s)=\frac{\sum_{k=0}^{m} a_k s^k}{\sum_{j=0}^{n} b_j s^j \prod_{i=1}^{\vartheta}(\tau_i s-1)(\tau_d s+1)} \tag{2}$$

2.1. The mirror mapping technique and CGSA

The H_∞ mixed sensitivity algorithm is a typical method to solve the H_∞ control problem. Fig. 2(a) gives the block diagram for the algorithm that is rearranged from the typical single feedback system Fig. 1(a) [29]. The generalized plant is expressed as Eq. (3), where $P_{11}=[W_1\ 0]^T$, $P_{12}=[-W_1\overline{G}\ W_2\overline{G}]^T$, $P_{21}=I, P_{22}=-\overline{G}$. The objective of H_∞ mixed sensitivity algorithm is to find a controller $\overline{C}(s)$ such that the generalized plant $P(s)$ is stabilized and $\|F_l(P,\overline{C})\|_\infty<\gamma$ in parallel. The constant γ represents the desired performance level of the closed loop system. Actually, $F_l(\cdot)$ denote the linear fractional transformation and $F_l(P,\overline{C})$ is the transfer function from $r \to z$. As shown in (4), $F_l\left(P,\overline{C}\right)$ is composed of the sensitivity function $S(s)$, the complementary sensitivity function $T(s)$ and the weight functions $W_1(s), W_2(s)$.

$$P(s)=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}=\begin{bmatrix} W_1 & -W_1\overline{G} \\ 0 & W_2\overline{G} \\ I & -\overline{G} \end{bmatrix} \tag{3}$$

$$F_l\left(P,\overline{C}\right)=\begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix}=\begin{bmatrix} W_1\left(I+\overline{G}\overline{C}\right)^{-1} \\ W_2\overline{G}\overline{C}\left(I+\overline{G}\overline{C}\right)^{-1} \end{bmatrix} \tag{4}$$

In the H_∞ mixed sensitivity algorithm, $S(s)$ and $T(s)$ optimized by the H_∞ optimization theory are characterized as follows: (1) The sensitivity function $S(j\omega)$ should be set as small as possible in the low frequency zone, to guarantee its capacities of low frequency interference rejection and following the reference signal. (2) In the high frequency zone, $S(j\omega)$ should be equal to 1 to preserve the actuator from its frequent action related to the high frequency noise. (3) W_2 is the supremum of the model uncertainty, with high-pass feature. Thus, $T(j\omega)$ should be low-pass such that $\|W_2T\|_\infty<1$. Based on the conclusion, CGSA is a simplified H_∞ mixed sensitivity algorithm by directly shaping the singular value curves of $S(s)$ and $T(s)$ [30], and there exists the correlativity $T(s)=I-S(s)$. As shown in Fig. 2(b), the complementary sensitivity function $T(s)$ of a typical control system has a low-pass characteristics to guarantee the robust performance, and the largest singular value equals to unit one to follow the reference signal without the tracking error. For the common stable plant $\overline{G}(s)$ in Fig. 1(a), the high-frequency asymptote slop of $T(s)$ in Fig. 2(b) is usually suggested to be -20 dB/dec, -40 dB/dec and -60 dB/dec, which determines how much the system is sensitive to the invalid disturbance frequency. That generates the common shaping