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Further improved stability criteria for uncertain T–S fuzzy systems with time-varying delay by (m, N)-delay-partitioning approach ☆

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ABSTRACT

This paper mainly focuses on the robust stability criteria for uncertain T–S fuzzy systems with time-varying delay by (m,N)-delay-partitioning approach. A modified augmented LKF is established by partitioning the delay in all integral terms. Via taking into account of (i) the relationship between each subinterval and time-varying delay and (ii) the independent upper bounds of the delay derivative in the various delay intervals, some new results on tighter bounding inequalities such as Peng-Park's integral inequality and Free-Matrix-based integral inequality are introduced to effectively reduce the enlargement in bounding the derivative of LKF as much as possible, therefore, significant less conservative results can be expected in terms of e_s and LMIs, which can be solved efficiently with the Matlab LMI toolbox. Furthermore, it is worth mentioning that, when the delay-partitioning number m is fixed, the conservatism is gradually reduced with the increase of another delay-partitioning number N, but without increasing any computing burden. Finally, two numerical examples are included to show that the proposed method is less conservative than existing ones.

1. Introduction

Since Takagi–Sugeno (T–S) fuzzy model was first introduced in [1], much effort has been made in the stability analysis and control synthesis of this model during the past three decades, due to the fact that it can combine the flexibility of fuzzy logic theory and fruitful linear system theory into a unified framework to approximate complex nonlinear systems, such as truck–trailer system, TORA system, inverted pendulum system, ball-and-beam system and chaotic systems [1–4]. On the other hand, as a source of instability and deteriorated performance, time-delay often occurs in many dynamic systems such as biological systems, chemical processes, communication networks and so on [5–9]. Therefore, in the last decade, stability analysis of T–S fuzzy systems with (time-varying) delay has been an active topic, see, e.g., [10–18] and references therein. Among the recent techniques adopted in this area, the delay-partitioning approach [19] should be the most worth-mentioning since it has been proven that less conservative results may be expected with the increase of delay-partitioning segments [20,16]. On the other hand, as [21] points out that, for delay-partitioning approach, further less conservative stability conditions for time-varying delay systems can be achieved by taking into account of independent upper bounds of the delay derivative in various delay-partitioning intervals. Furthermore, as is well-known, it is necessary to take the derivative of the LKF to derive a stability condition, and the main difficulty lies in bounding the integrals that appear in the derivative. Among the new techniques adopted in bounding integral, the most noteworthy is the Free-Matrix-based integral inequality (FMII) [22] that includes the Wirtinger inequality as a special case. Since the FMII is composed of a set of adjustable slack variables, it can provide extra freedom in reducing the conservativeness of the inequality and further lead to less conservative conditions than the use of the Wirtinger-based inequality does [22].

Recently, by non-uniformly dividing the whole delay interval into multiple segments and choosing different Lyapunov functionals to different segments, [23] has established less

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conservative delay-derivative-dependent stability criteria than those in [24,13] in a convex way for the uncertain T–S fuzzy systems with interval time-varying delay. Very recently, via delay-partitioning approach and Peng-Park's integral inequality established by the reciprocally convex combination (RCC) technique [25], [20] has developed less conservative stability criteria than those in [26,11,27] for the uncertain T–S fuzzy systems with interval time-varying delay. More recently, based on a novel LKF and the RCC technique, [16] has achieved less conservative results than those in [20,11,15,12–14] via introducing some fuzzy-weighting matrixes to express the relationship of the T–S fuzzy models. Most recently, by delay-partitioning approach, Finsler's lemma and a modified LKF, [17] has employed Peng-Park's integral inequality [25] and Seuret-Wirtinger's integral inequality [28] to effectively bound the derivative of LKF and achieved less conservative stability criteria than those in [11,16,13] for the uncertain T–S fuzzy systems with time-varying delay.

However, when revisiting this problem, we find that the aforementioned works still leave plenty of room for improvement since (i) the relationship between time-varying delay and each subinterval is not taken into account in [23,20,16,17]; (ii) independent upper bounds of the delay derivative in various delay-partitioning intervals, and the relationships between the augmented state vectors $[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \ldots, x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}}$ are both ignored in [23,20,16]; (iii) Wirtinger-based inequality [28] is employed to bound the derivative of LKF in [16,17], nevertheless, the Wirtinger-based inequality cannot find further tighter upper bound because its parameters are not adjustable, which inevitably yields some unnecessary conservativeness in [16,17].

Motivated by the above-mentioned discussion, this paper aims to develop further improved stability criteria for uncertain T–S fuzzy systems with time-varying delay by the (m,N)-delay-partitioning approach and Finsler's lemma. The main contributions of this paper lie in the following aspects: firstly, the (m,N)-delay-partitioning approach partitions the time-varying delay $\tau(t)$ and its upper bound separately; secondly, an appropriate augmented LKF is established by partitioning the delay in all integral terms, and the $\tau(t)$-dependent/$\rho(t)$-dependent/$[X_{ij}]_{m\times m}$-dependent sub-LKFs are introduced to the augmented LKF, therefore, (i) the relationship between each subinterval and time-varying delay, (ii) the independent upper bounds of the delay derivative in the various delay intervals and (iii) the relationships between the augmented state vectors $[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \ldots, x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}}$ have been simultaneously taken a full consideration; thirdly, some new results on tighter bounding inequalities such as Peng-Park's integral inequality and FMII (which yields less conservative stability criteria than the use of Wirtinger-based inequality does) have been employed to reduce the enlargement in bounding the derivative of LKF as much as possible, therefore, less conservative results can be achieved in terms of e_s and LMIs; fourthly, it is worth mentioning that, when the delay-partitioning number m is fixed, the conservatism is gradually reduced with the increase of another delay-partitioning number N, but without increasing any computing burden. Finally, two numerical examples are included to show the effectiveness and the benefits of the proposed method.

The rest of this paper is organized as follows. The main problem is formulated in Section 2 and less conservative stability criteria for the uncertain T–S fuzzy systems with time-varying delay are derived in Section 3. In Section 4, two numerical examples are provided; and a concluding remark is given in Section 5.

Notations: Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices; the notation $A>(\geq)B$ means that $A-B$ is positive (semi-positive) definite; I (0) is the identity (zero) matrix with appropriate dimension; A^{T} denotes the transpose; $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$; "$*$" denotes the elements below the main diagonal of a symmetric block matrix; $C([-\tau,0],\mathbb{R}^n)$ is the family of continuous functions ϕ from interval $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_\tau=\sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$; let $x_t(\theta)=x(t+\theta),\ \theta\in[-\tau,0]$. In addition, because many abbreviations are used in this paper, they are given in Table 1 for the convenience of the reader.

2. Problem formulation

In this section, a class of uncertain T–S fuzzy system with time-varying delay is concerned. For each $i=1,2,\ldots,r$ (r is the number of plant rules), the ith rule of this T–S fuzzy model is represented as follows:

Plant rule i**:IF** $\theta_1(t)$ **is**M_{i1}, $\theta_2(t)$ **is** M_{i2}, $\cdots$, $\theta_p(t)$**is**M_{ip},THEN

$$\begin{cases}\dot{x}(t)=[A_i+\Delta A_i(t)]x(t)+[A_{di}+\Delta A_{di}(t)]x(t-\tau(t)), & t\geq 0\\ x(t)=\phi(t), & t\in[-\tau,0],\end{cases}\tag{1}$$

where $\theta_1(t)$, $\theta_2(t)$, $\cdots$, $\theta_p(t)$ are the premise variables, and each $M_{il}(i=1,2,\ldots,r; l=1,2,\ldots,p)$ is a fuzzy set. $x(t)\in\mathbb{R}^n$ is the state vector; $\phi(t)\in C([-\tau,0],\mathbb{R}^n)$ is the initial function; A_i and A_{di} are constant real matrices with appropriate dimensions; the delay, $\tau(t)$, is a time-varying functional satisfying

$$0\leq\tau(t)\leq\tau,\tag{2}$$

$$\dot{\tau}(t)<\mu,\tag{3}$$

where τ and μ are constants; the matrices $\Delta A_i(t)$ and $\Delta A_{di}(t)$ denote the uncertainties in the system and are defined as

$$[\Delta A_i(t),\Delta A_{di}(t)]=HF(t)[E_i,E_{di}],\tag{4}$$

where H, E_i and E_{di} are known constant matrices and $F(t)$ is an unknown matrix function satisfying

$$F^{\mathrm{T}}(t)F(t)\leq I.\tag{5}$$

By a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model in (1) can be represented by

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{r}h_i(\theta(t))\{A_i(t)x(t)+A_{di}(t)x(t-\tau(t))\},\\ x(t)=\phi(t), \quad t\in[-\tau,0],\end{cases}\tag{6}$$

where $A_i(t)=A_i+\Delta A_i(t)$, $A_{di}(t)=A_{di}+\Delta A_{di}(t)$ and

$$h_i(\theta(t))=\frac{\prod_{l=1}^{p}M_{il}(\theta_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p}M_{il}(\theta_l(t))},\quad i=1,\ldots,r,\tag{7}$$

Table 1
Abbreviations and their definitions.

Abbreviation	Definition
LKF	Lyapunov–Krasovskii functional
LMIs	Linear matrix inequalities
He(A)	the sum of matrices A and A^{T}
FMII	Free-Matrix-based integral inequality
MAUB	Maximum admissible upper bound
RCC	Reciprocally convex combination
(m,N)	see Definition 1
$\rho(t)$	see (9)
e_s	see (12)
$[X_{ij}]_{m\times m}$	$\begin{bmatrix}X_{11} & \cdots & X_{1m}\\ \vdots & \ddots & \vdots\\ * & \cdots & X_{mm}\end{bmatrix}$