ISA Transactions 59 (2015) 79–84

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

Parameter identification of fractional order linear system based on Haar wavelet operational matrix

Yuanlu Li [a,b,*,1], Xiao Meng [b,1], Bochao Zheng [a,b,1], Yaqing Ding [b,1]

[a] B-DAT, School of Information and Control, Nanjing University of Information Science & Technology, Nanjing 210044, China
[b] Jiangsu Collaborative Innovation Center on Atmospheric Environment and Equipment Technology, Nanjing University of Information Science & Technology, Nanjing 210044, China

ARTICLE INFO

Article history:
Received 23 February 2015
Received in revised form
9 July 2015
Accepted 20 August 2015
Available online 4 September 2015
This paper was recommended for publication by Mohammad Haeri.

Keywords:
Fractional order system
System identification
Haar wavelet
Operational matrix

ABSTRACT

Fractional order systems can be more adequate for the description of dynamical systems than integer order models, however, how to obtain fractional order models are still actively exploring. In this paper, an identification method for fractional order linear system was proposed. This is a method based on input–output data in time domain. The input and output signals are represented by Haar wavelet, and then fractional order systems described by fractional order differential equations are transformed into fractional order integral equations. Taking use of the Haar wavelet operational matrix of the fractional order integration, the fractional order linear system can easily be converted into a system of algebraic equation. Finally, the parameters of the fractional order system are determined by minimizing the errors between the output of the real system and that of the identified system. Numerical simulations, involving integral and fractional order systems, confirm the efficiency of the above methodology.

1. Introduction

A fractional order system (FOS) is a system that is modeled by a fractional differential equation containing derivatives of non-integer order. Recently, considerable attention has been paid to the FOS. The reason for this is because a growing number of physical systems can be compactly described using FOS, such as the semi-infinite lossy (RC) transmission line [1,2], diffusion of heat into semi-infinite solid [2], viscoelastic materials [3,4], electrochemical processes [5], dynamics of porous media [6], continuous time random walk [7]. In addition, theoretical and experimental results have been shown that the fractional order controller had better dynamic responses and more robustness to model uncertainties in comparison with the classical controllers [8].

However, because the geometric and physical interpretation of fractional calculus is not as distinct as integer calculus, it is difficult to model real systems as FOS directly based on mechanism analysis. Therefore, system identification is a practical way to model a FOS. For integral order system (IOS), once the maximum order of the system to be identified is determined, the parameters of the model can be optimized directly. However, for a FOS, because identification requires the choice of the number of fractional order operators, the fractional order of the operators, and finally the coefficients of the operators, the identification process of a FOS is more complex than that of an IOS [9]. Most classical identification methods cannot directly applied to identification of a FOS.

Existing identification of a FOS can be mainly divided into two categories: time-domain system identification and frequency-domain system identification. In time domain [10–12], the parameters of a system to be identified are determined by minimizing the error between the output of the actual system and that of the identified system. For instance, Poinot and Trigeassou [13,14] have used fractional models to identify thermal systems, Sabatier et al. used the identified fractional order model to estimate the crankability of lead-acid batteries [15]. Compared to integer order system, the most obvious difference lies in identification of the fractional order of the operators. Therefore nonlinear optimization method has adopted to identify the order of a FOS [16,17]. Moreover, some intelligent algorithm were also applied for identification of FOS, such as genetic algorithms [18,19], differential evolution algorithm [20], particle swarm optimization [21,22]. In frequency domain, Li et al. [23] used the least squares method to investigate the frequency response identification technique. Nazarian et al. [24] developed an identification method of FOS according to input output frequency contents. Hartley et al. [9] discussed an identification method for FOS using continuous order-distributions. Besides above mentioned methods, recently, a refined instrumental variable method for continuous-time systems

* Corresponding author.
E-mail addresses: lyl_nuist@nuist.edu.cn (Y. Li), 1725932714@qq.com (X. Meng), zhengbochao81@126.com (B. Zheng), dingyaqing@163.com (Y. Ding).
[1] Tel.: +86 2558731276.

http://dx.doi.org/10.1016/j.isatra.2015.08.011
0019-0578/

was extended to identify FOS [25], subspace method was proposed to identify a continuous-time FOS. In the proposed method, the parameter matrices were identified using the subspace-based technique and the commensurate orders were determined by using nonlinear programming [26], fractional Laguerre basis was proposed to identify FOS [27]. So, how to identify the FOS is still an open problem.

Operational matrix has been widely used to deal with FOS. The main characteristic behind the approach is that it converts these problems to those of solving a system of algebraic equation thus greatly simplifying the problem. Typical examples are the block-pulse functions [12,28], the Jacobi operational matrix of fractional integration [29–31], Legendre polynomials [32–34], Chebyshev polynomials [35,36] and Haar wavelets [37,38].

Main aim of this paper is to use the Haar wavelet operational matrix to identify the FOS. The input and output signals are represented by Haar wavelets, and then the FOS described by fractional order differential equations are converted into fractional order integral equations. Taking use of the Haar wavelet operational matrix of the fractional order integration, the FOS can easily be converted into a system of algebraic equation. The parameters of the FOS are determined by minimizing the error between the output of the real system and that of the identified system.

The organization of this paper is as follows: in Section 2, the fractional calculus, FOS and problem statement is introduced. In Section 3, the identification method based on Haar wavelet operational matrix is proposed. And verification of the method is provided in Section 4. Finally, conclusions are made in Section 5.

2. Fractional order system

2.1. The definition of fractional calculus

There are several definitions for the general fractional differentiation and integration, such as the Grünwald–Letnikov definition, the Riemann–Liouville definition and Caputo definition [39]. Here the Riemann–Liouville fractional integral and Caputo fractional derivative were given as following, which will be used in this paper.

The Riemann–Liouville fractional integration of order $\alpha > 0$ is defined as

$$(I^{\alpha}f)(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}f(\tau)d\tau = \frac{1}{\Gamma(\alpha)}t^{\alpha-1}*f(t)H(t) \tag{1}$$

where Γ is the Gamma function, $H(t)$ is a Heaviside function.

When the Riemann–Liouville derivative was used to model real-world phenomena, initial conditions with fractional order derivative are difficult to obtain. So we introduce a modified fractional differential operator D^{α} proposed by Caputo,

$$(D^{\alpha}f)(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau, \tag{2}$$

where $n-1 < \alpha < n$ and n is an integer.

The relation between the Riemann–Liouville integral and Caputo derivative is given by the following expressions:

$$(I^{\alpha}D^{\alpha}f)(t) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0^+)\frac{t^k}{k!}, \tag{3}$$

where $n-1 < \alpha < n$ and n is an integer.

2.2. Fractional order systems

A fractional order system (FOS) is a system that is modeled by a fractional differential equation containing derivatives of non-integer order. A single input single output (SISO) linear time invariant (LTI) system may be described by the following fractional order differential equation:

$$\sum_{i=0}^{n} a_i D^{\alpha_i} y(t) = \sum_{j=0}^{m} b_j D^{\beta_j} f(t) \tag{4}$$

Under the zero initial conditions, applying the Laplace transform to Eq. (4) the input–output representation of the FOS can be written in the form of a transfer function:

$$G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{5}$$

where α_i and β_j are arbitrary real positive, $f(t)$ and $y(t)$ are the input and output of the system, respectively.

3. Identification method based on the Haar wavelet operational matrix

3.1 Haar wavelet

If $n = 2^j + k$, where k and j are integers and $j \ge 1$ and $0 \le k \le 2^j$, then $h_n = h_1(2^j t - kb)$. $h_0(t) = 1$ if $0 \le t < b$ and 0 otherwise, $h_1(t) = \begin{cases} 1, & 0 \le t < b/2 \\ -1, & b/2 \le t < b. \end{cases}$ and 0 otherwise.

An arbitrary signal $x(t) \in L^2[0, b]$ can be expanded by Haar wavelet, i.e.,

$$x(t) = \sum_{i=0}^{\infty} c_i h_i(t), \tag{6}$$

where the Haar coefficients c_i, $i = 0, 1, 2, \cdots$, are determined by

$$c_i = 2^j \int_0^b x(t)h_i(t)dt \tag{7}$$

In practice, only the first N terms of Eq. (6) are considered, where N is a power of 2. So we have

$$x(t) \approx \sum_{i=0}^{N-1} c_i h_i(t) = C_N{}^T H_N(t) = \hat{x}(t) \tag{8}$$

where the superscript T indicates transposition, the Haar coefficient vector C_N and the Haar function vector $H_N(t)$ are defined as

$$C_N \triangleq [c_0, c_1, \ldots, c_{N-1}]^T, \tag{9}$$

$$H_N(t) \triangleq \left[h_0(t), h_1(t), \ldots, h_{N-1}(t)\right]^T. \tag{10}$$

Taking suitable collocation points as following

$$t_i = \frac{(2i-1)b}{2N}, \quad i = 1, 2, \ldots, N, \tag{11}$$

The N-square Haar matrix $\Psi_{N\times N}$ can be defined by

$$\Psi_{N\times N} \triangleq \left[H_N\left(\frac{1}{2N}b\right) \; H_N\left(\frac{3}{2N}b\right) \; \ldots \; H_N\left(\frac{2N-1}{2N}b\right) \right]. \tag{12}$$

3.2. Block pulse operational matrix of the fractional order integral

N-term Block pulse functions are defined as following

$$\varphi_i(t) = \begin{cases} 1 & ib/N \le t < (i+1)b/N \\ 0 & \text{otherwise} \end{cases}, \; i = 0, 1, 2, \cdots, (N-1), \tag{13}$$