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Research Article

Finite-time stabilization of uncertain nonholonomic systems in feedforward-like form by output feedback

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ABSTRACT

This paper investigates the problem of finite-time stabilization by output feedback for a class of nonholonomic systems in chained form with uncertainties. Comparing with the existing relevant literature, a distinguishing feature of the systems under investigation is that the x-subsystem is a feedforward-like rather than feedback-like system. This renders the existing control methods inapplicable to the control problems of the systems. A constructive design procedure for output feedback control is given. The designed controller renders that the states of closed-loop system are regulated to zero in a finite time. Two simulation examples are provided to illustrate the effectiveness of the proposed approach.
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1. Introduction

In the past decades, the control of nonholonomic systems has received significant amount of interests from researchers worldwide because they can model many frequently met mechanical systems, such as mobile robots, car-like vehicle and under-actuated satellites. However, due to the limitation imposed by Brockett's necessary condition [1], this class of nonlinear systems cannot be stabilized by stationary continuous state feedback. To overcome this difficulty, with the effort of many researchers a number of intelligent approaches have been proposed, which can mainly be classified into discontinuous time-invariant stabilization [2,3], smooth time-varying stabilization [4–6] and hybrid stabilization [7,8], see the survey paper [9] and references therein for more details. Mainly thanks to these valid approaches, the robust issue of nonholonomic systems has been well-studied and a number of interesting results have been established over the last years, for example, one can see [10–20] and the references therein.

As is well-known, stability is one of the most important research topics since it is the precondition for the system to work normally [21–23]. There is a special kind of stability, finite-time stability, that is, the solution of an asymptotically stable system reaches the equilibrium point in finite time. Compared with the commonly used notion of asymptotic stability, the finite-time stable systems have many nice features such as faster convergence rates, higher accuracies and better disturbance rejection properties [24]. As a consequence, the finite-time stability and stabilization have attracted increasing attention over the last years [25–30]. Particularly, by using state feedback, the authors in [31] addressed the finite-time stabilization of nonholonomic systems with weak drifts. The works [32–35] further extended the results in [31] to the nonholonomic systems with uncertain parameters and perturbed terms. It should be noted that all above papers are concerned with the systems in feedback-like form (i,e., the x-subsystem of considered systems is a feedback-like system), and thus these methods are not applicable to nonholonomic systems in feedforward-like form. Questions naturally arise: *is it possible to finite-time stabilize nonholonomic systems in feedforward-like form? If possible, under what conditions can we design such controllers and how?* To our best knowledge, in the literature there have not been results which provide answers to these questions.

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Motivated by the above discussion and the example presented in Section 2.1, this paper focuses on solving the finite-time stabilization problem for a class of nonholonomic feedforward systems using output feedback. The contributions are highlighted as follows: (i) the output feedback stabilization problem of the nonholonomic systems, x-subsystem of which is feedforward-like form, is studied for the first time. (ii) A sufficient condition on characterizing the nonlinear growth of the nonholonomic feedforward systems for its finite-time stabilization is derived. (iii) Based on a novel switching strategy to overcome the obstacle that the discontinuous change of coordinates is inapplicable to the finite-time output feedback control of nonholonomic systems, and by skillfully using the homogeneous domination approach, a systematic output feedback control design procedure is proposed to render the states of closed-loop system to zero in a finite time. (iv) An application example for hopping robot is modeled and solved by the proposed method.

Notations: Throughout this paper, the following notations are adopted. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. For a given vector X, X^T denotes its transpose, and $|X|$ denotes its Euclidean norm. C^i denotes the set of all functions with continuous ith partial derivatives. K denotes the set of all functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; K_∞ denotes the set of all functions which are of class K and unbounded. Besides, the arguments of the functions will be omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f.

2. Motivating example and problem formulation

2.1. Motivating example

Consider a kinematic hopping robot, as shown in Fig. 1. As cited in [4], this robot consists of a body with an actuated leg that can rotate and extend; the constraint on the system is conservation of the angular momentum. Let (ψ, l, θ) be the angle, leg extension, and leg angle of the robot. For the sake of simplicity we take the body mass equal to one and concentrate the mass m of the leg at the foot. The upper leg length is also taken equal to one and l represents the extension of the leg. The angular momentum of the robot is given by

$$\dot{\theta} + m(l+1)^2(\dot{\theta} + \dot{\psi}) = 0 \tag{1}$$

Since we the leg angle and extension can be controlled directly, we choose their velocities as our inputs. Thus, the kinematics of the robot can be expressed as

$$\begin{aligned} \dot{\psi} &= u_0 \\ \dot{\theta} &= -\frac{m(l+1)^2}{1+m(l+1)^2} u_0 \dot{l} = u_1 \end{aligned} \tag{2}$$

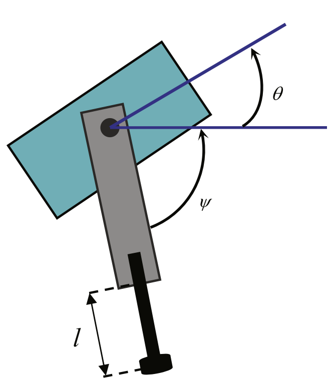

Fig. 1. A simple hopping robot.

Note that the second equation is a consequence of conservation of angular momentum. We expand the equation using a Taylor series about $l=0$ and obtain

$$\dot{\theta} = -\frac{m}{1+m}\dot{\psi} - \frac{2m}{(1+m)^2} l u_0 - f(l) u_0 \tag{3}$$

For system (2), by taking the following state transformation:

$$x_0 = \psi, \quad x_1 = -\theta - \frac{m}{1+m}\psi, \quad x_2 = l \tag{4}$$

we obtain

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= \frac{2m}{(1+m)^2} x_2 u_0 + f(x_2) u_0 \\ \dot{x}_2 &= u_1 \end{aligned} \tag{5}$$

It is evident that the x-subsystem of system (5) has a feedforward-like rather than feedback-like structure, that is, system (5) is a nonholonomic system in feedforward-like form (nonholonomic feedforward system). Due to the special structure, it is easily verified that the existing finite-time control methods are inapplicable to the system (5). Therefore, an interesting problem is how to design a finite-time stabilizer for the system (5) and more general nonholonomic feedforward systems. In this paper, we will focus our attention on solving this problem.

2.2. Problem statement and preliminaries

In this paper, we consider the finite-time output feedback stabilization for the following class of nonholonomic systems in feedforward-like form:

$$\begin{aligned} \dot{x}_0 &= d_0 u_0 + \phi_0(t, x_0) \\ \dot{x}_1 &= d_1 x_2 u_0 + \phi_1(t, x_2, \ldots, x_n, u_0, u_1) \\ \dot{x}_2 &= d_2 x_3 u_0 + \phi_2(t, x_3, \ldots, x_n, u_0, u_1) \\ &\vdots \\ \dot{x}_{n-1} &= d_{n-1} x_n u_0 + \phi_{n-1}(t, x_n, u_0, u_1) \\ \dot{x}_n &= d_n u_1 \\ y &= (x_0, x_1)^T \end{aligned} \tag{6}$$

where $(x_0, x)^T = (x_0, x_1, \ldots, x_n)^T \in R^{n+1}$, $u = (u_0, u_1)^T \in R^2$, $y \in R^2$ are the system state, control input and system output, respectively; d_i's are disturbed virtual control coefficients; and ϕ_i's denote the input and states driven uncertainties, which are called as the nonlinear drifts of the system (6).

Remark 2.1. Although great progress on nonlinear feedforward systems has been made [36,38–40], for nonholonomic feedforward system (6), how to construct a finite-time stabilizer via output feedback is still very difficult problem. The crucial obstacle mainly comes from two aspects. One is that time-varying coefficient $d_i u_0$ makes the x-subsystem uncontrollable in the case of $u_0 = 0$. The other one is that, the discontinuous change of coordinates, as a common method for control design of nonholonomic systems, is inapplicable to the finite-time output feedback control of nonholonomic systems. Therefore, how to overcome these obstacles and design a

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