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Research Article

New passivity criteria for memristive uncertain neural networks with leakage and time-varying delays ☆

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ABSTRACT

In this paper, the problem of passivity analysis is studied for memristor-based uncertain neural networks with leakage and time-varying delays. By combining differential inclusions with set-valued maps, the system of memristive neural networks is changed into the conventional one. By adding a triple quadratic integral and relaxing the requirement for the positive definiteness of some matrices, a proper Lyapunov–Krasovskii functional is constructed. Based on the establishment of the novel Lyapunov–Krasovskii functional, the new passivity criteria are derived by mainly applying Wirtinger-based double integral inequality, S-procedure and so on. Moreover, the conservatism of passivity conditions can be reduced. Finally, four numerical examples are given to show the effectiveness and less conservatism of the proposed criteria.

1. Introduction

It is well known that the neural networks are so important that they have been widely applied in various areas such as signal processing, pattern recognition, optimization problems and so on. For the recent application, it mainly concentrates upon promoting the stability conditions of the neural networks, because time delays cannot be eliminated and they often bring about instability. Thus, in order to apply neural networks with high quality, stability analysis develops more and more significantly. Many relevant results have been reported in the literature [1–15]. Meanwhile, many researchers have investigated many dynamic behaviors which are closely related to stability such as H_∞ control [25,37,38], state estimation [29,32,33,36] and so on.

On one hand, as mentioned in [10], it is recognized that most of the recent results for studying the dynamic systems are constructed on the basis of Lyapunov–Krasovskii functional method. Based on the theory of Lyapunov–Krasovskii functional method, the conservatism of the results is one key factor to judge whether delay-dependent criteria are better or not. To derive less conservative results, one should know the reason why the conservatism of the Lyapunov–Krasovskii functional method exists. Generally speaking, there are two main indexes: one index is the construction of Lyapunov–Krasovskii functional and the other is the bound on its derivative. Therefore, it is so crucial for constructing a proper Lyapunov–Krasovskii functional as to obtain the less conservative criteria. In order to establish a suitable Lyapunov–Krasovskii functional, there are two suggested ways: one is to expand the Lyapunov–Krasovskii functional by adding a delay term to the state vector and the other is to construct Lyapunov–Krasovskii functional by partitioning the delay into many segments. Besides, it is necessary to manipulate the derivative of Lyapunov–Krasovskii functional to obtain the conservative criteria. While taking the derivative of Lyapunov–Krasovskii functional, the bounds of the integrals emerge in the derivative. Two main techniques are usually used for solving such integrals: the integral inequality method and the free-weighting method. Usually, free-weighting matrices can mainly be added via the introduction of zero inequality and S-procedure. Very recently, there is a new integral inequality named Wirtinger-based

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double integral inequality which can be applied in obtaining less conservative criteria in comparison with the Jensen inequality. How to construct a proper Lyapunov–Krasovskii functional by adding some delay terms to the state vector such as a triple quadratic integral is still an open question. Meanwhile, how to obtain less conservative results by effectively applying S-procedure and Wirtinger-based double integral inequality in the novel Lyapunov–Krasovskii functional is still an interesting challenge.

On the other hand, it is well-known that the passivity theory plays an important role in the analysis of the stability of dynamical systems, nonlinear control and other areas. The characteristic of passive properties is that they can make the systems internally stable. During the past several years, the passivity problem for time-delay systems has been investigated in the literature [16–22]. In [16–22], authors investigated the passivity of neural networks with time-varying delays and gave the corresponding criteria for checking the passivity. It is worth noting that the passivity conditions in [16–22] were derived on the basis of quadratic Lyapunov–Krasovskii functional in which the involved symmetric matrix was always assumed to be positive. However, the results in [16–22] may still be conservative and not relaxed enough to be applied. Thus, it is necessary to investigate whether the results in [16–22] could be improved by constructing relaxed conditions. Recently, in [23], authors revisited the problem of passivity analysis for neural networks by giving relaxed passivity conditions.

However, it is worth pointing out that the given criteria in [16–23] have been based on the following assumptions: firstly, the time-varying delays are continuously differentiable; secondly, the derivative of time-varying delay is bounded and is smaller than one; thirdly, the activation functions are monotonically nondecreasing. Actually, time delays can occur in an irregular fashion and sometimes the time-varying delays are not differentiable. So those above assumptions are not milder enough to be extensively applied. In addition, recently, many researchers have paid close attention to a state-dependent switching system named memristor-based neural networks whose connection weights vary according to the changes of their state (see [39–50]). Particularly, many dynamic behaviors such as stability [50], synchronization [42,43,46], passivity [44,45,49], state estimation [47], dissipativity [48] and so on have been studied by many scholars. However, the above-mentioned memristive neural networks rarely involved problem caused by both parameter uncertainties and leakage delay. It is well-known that the parameter uncertainties are inherent features of many physical systems and may cause the instability and poor performance. These uncertainties may arise because of the variations in system parameters, modeling errors or some ignored factors [24–35]. Moreover, since time delay in the stabilizing negative feedback term has a tendency to destabilize a system [24,26,27,32–35], the leakage delay also plays a great impact on the dynamics of neural networks. Therefore, it is necessary to study the passivity problem for memristor-based neural networks with parameter uncertainties and leakage delay.

Motivated by the above discussions, the relaxed passivity conditions are adopted for memristive uncertain neural networks with leakage and time-varying delays in this paper. The main contribution of this paper lies in the following five aspects: firstly, the studied memristive neural networks contain not only leakage delay but also parameter uncertainties; secondly, the new Lyapunov–Krasovskii functional is constructed by introducing a triple quadratic integral and relaxing the restriction on the positive definiteness of some matrices; thirdly, to solve the time-derivative of the triple integral, Wirtinger-based double integral inequality is well applied in researching the lower bound of the corresponding double integral; fourthly, to relax the revelent conditions about the positive definiteness of some matrices, Jensen inequality is fully combined with Schur complement theory to prove that the corresponding Lyapunov–Krasovskii functional $V(t)$ must be positive definite; finally, the obtained sufficient conditions require neither the differentiability of time-varying delays nor the bound or the monotony of the activation functions. Four examples are given to show the effectiveness and less conservatism of the proposed criteria in the end.

Notations: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively. $[\cdot,\cdot]$ represents the interval. $Q>0$ $(Q\geq 0, Q<0, Q\leq 0)$ means that the matrix Q is symmetric positive definite (positive-semi definite, negative definite and negative-semi definite). $\|\cdot\|$ refers to the Euclidean vector norm. $\mathbb{R}^n$ denotes n-dimensional Euclidean space. $\mathcal{C}([-\rho,0],\mathbb{R}^n)$ represents Banach space of all continuous functions. $\mathbb{R}^{m\times n}$ is the set of $m\times n$ real matrices. $*$ denotes the symmetric block in symmetric matrix. For matrices $\mathcal{S}=(s_{ij})_{m\times n}$, $\mathcal{T}=(t_{ij})_{m\times n}$, $\mathcal{S}\gg\mathcal{T}(\mathcal{S}\ll\mathcal{T})$ means that $s_{ij}\gg t_{ij}(s_{ij}\ll t_{ij})$, for $i=1,2,\ldots,m, j=1,2,\ldots,n$. And by the interval matrix $[\mathcal{S},\mathcal{T}]$, it follows that $\mathcal{S}\ll\mathcal{T}$. For $\forall\mathcal{V}=(v_{ij})_{m\times n}\in[\mathcal{S},\mathcal{T}]$, it means $\mathcal{S}\ll\mathcal{V}\ll\mathcal{T}$, i.e, $s_{ij}\ll v_{ij}\ll t_{ij}$ for $i=1,2,\ldots,m, j=1,2,\ldots,n$. $co\{\Pi_1,\Pi_2\}$ denotes the closure of the convex hull generated by real numbers Π_1 and Π_2. Let $\overline{c}_i=\max\{\dot{c}_i,\ddot{c}_i\}$, $\underline{c}_i=\min\{\dot{c}_i,\ddot{c}_i\}$, $\overline{a}_{ij}=\max\{\dot{a}_{ij},\ddot{a}_{ij}\}$, $\underline{a}_{ij}=\min\{\dot{a}_{ij},\ddot{a}_{ij}\}$, $\overline{b}_{ij}=\max\{\dot{b}_{ij},\ddot{b}_{ij}\}$, $\underline{b}_{ij}=\min\{\dot{b}_{ij},\ddot{b}_{ij}\}$. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem statement and preliminaries

In this section, a general class of memristive neural networks is introduced as follows:

$$
\begin{aligned}
&\dot{x}_i(t)=-\tilde{c}_i(x_i(t))x_i(t-\delta)+\sum_{j=1}^{n}\tilde{a}_{ij}(x_i(t))f_j(x_j(t))+\sum_{j=1}^{n}\tilde{b}_{ij}(x_i(t))f_j(x_j(t-\tau_j(t)))+u_i(t),\quad t\geq 0,\ \ i=1,2,\ldots,n,\\
&y_i(t)=f_i(x_i(t)),\quad t\geq 0,\ \ i=1,2,\ldots,n,\\
&x_i(t)=\phi_i(t),\quad t\in[-\rho,0],\ \ \rho=\max\{\delta,\tau\},
\end{aligned}
\tag{1}
$$

where $x_i(t)$ stands for the neuron state vector of the system, $f_i(x_i(t))\in\mathbb{R}^n$ and $f_i(x_i(t-\tau_i(t)))\in\mathbb{R}^n$ are the nonlinear activation function without and with time-varying delay, respectively, $u_i(t)$ is the input vector, $y_i(t)\in\mathbb{R}^n$ is the output vector of the networks. δ is the leakage delay and $\tau(t)$ is the discrete time-varying delay. They satisfy the following conditions: $\delta\geq 0$, $0\leq\tau(t)\leq\tau(\delta$ and τ are constants). Besides, $\phi_i(t)$ is the initial condition and is bounded and continuously differential on $[-\rho,0]$. $\tilde{c}_i$ describes the rate with which each neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs. $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ represent the element of the connection weight matrix and the discretely delayed connection weight matrix, respectively. They satisfy the following conditions:

$$
\tilde{c}_i(x_i(t))=\begin{cases}\dot{c}_i, & |x_i(t)|\leq H_i,\\ \ddot{c}_i, & |x_i(t)|>H_i,\end{cases}
$$