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# Admissibility analysis for discrete-time singular systems with randomly occurring uncertainties via delay-divisioning approach

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A B S T R A C T

This paper deals with the problem of admissibility analysis for discrete-time singular system with time-delays. The uncertainties occurring in the system parameters are assumed to be random. By constructing Lyapunov functional, sufficient delay-dependent stochastic admissibility conditions are established via delay divisioning approach in terms of linear matrix inequalities (LMIs), which can be easily checked by utilizing the numerically efficient Matlab LMI toolbox. Numerical examples and their simulation results are given to illustrate the effectiveness of the obtained theoretical results.



## 1. Introduction

Singular systems, which are also known as descriptor systems, semi-state-space systems, implicit systems, differential algebraic systems and generalized state-space systems are dynamic systems whose behavior is described by both differential equations (or difference equations) and algebraic equations. These systems can preserve the structure of practical systems and have extensively applied in electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large-scale systems, interconnected systems, economics, optimization problems, feedback systems, robotics, biology, etc., for details reader may refer [5,6,23] and references therein. Study of singular systems is much more complicated than that of standard state-space time-delay systems, since not only stability condition should obtained, but also the system regularity and causality (for discrete systems), or impulse-free condition (for continuous systems) need to be guaranteed. Also, time-delay frequently encountered in various practical systems and is often the cause for instability and poor performance. Dynamics of the singular systems would be more complex since time-delays may exist in the differential equations and/or algebraic equations. During the past few years, much attention has been devoted to admissibility problem for singular time-delay systems [7,10]. It is worth pointing out that the time-delays in some systems often exist in a stochastic manner, and its probabilistic characteristic, such as Binomial distribution or normal distribution, can regularly be obtained by using the statistical methods [1,20].

On the other hand, parameter uncertainties are unavoidable due to the modeling inaccuracies, variations of the operating point, aging of the devices, etc. Thus, the issue of robust analysis has taken into account in all sorts of systems by many researchers [25,31]. Randomly occurring uncertainties (ROUs) are a new type of uncertainties proposed in Ma et al. [18] due to the fact that the uncertainties may subject to random changes in environment circumstances, for instance, repairs of components and sudden environmental disturbances. These uncertainties may occur in a probabilistic way with certain types and intensity [11,26].

Dynamical systems are usually affected by external perturbations which can be treated as stochastic inputs to the system. It is proved that certain stochastic inputs have the tendency to destabilize the system. In order to view a realistic behavior of the system, a degree of randomness must be incorporated into the model. Hence, the stability problems for stochastic systems become important to investigate in both the continuous-time and discrete-time frameworks [27]. The stability problem for the singular Markovian jump systems with time-delay studied in many works [2,19,3,28]. In Lin et al. [15] and Ma and Boukas [17], the robust $H_\infty$ filtering problem for discrete-time singular Markovian

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jump systems with mode-dependent time-delay and parameter uncertainties was discussed. In Jarina Banu and Balasubramaniam [12], authors have dealt with the admissibility problem for discrete-time nonlinear systems via Takagi–Sugeno fuzzy approach. It is worth noticing that various methods were developed to obtain less conservative results. Results produced by delay decomposition method and delay partitioning method are much less conservative than the existing ones. These methods have been applied to regular delay systems [9,33–35]. Moreover, based on this approach, a few results on admissibility analysis for singular time-delay systems were investigated [8,24,36].

While dealing with delay-dependent stability analysis, one of the key for testing the conservatism of the results is to find a maximum delay bound for which the concerned system ensures the required result. The major issue in the delay-dependent stability analysis is how to reduce the possible conservatism. The less conservativeness in Lyapunov stability theory heavily depends on the introduction of the Lyapunov–Krasovskii functional when dealing with time-delays. Thus, there is still room open for further development of some new techniques and approaches to improve the feasibility region of stability. Moreover, to the best of authors knowledge, the stochastic admissibility problem for discrete-time singular systems with ROUs and nonlinear disturbances is not fully investigated.

Inspired by the above works, in this paper, authors revisit the admissibility condition of discrete-time singular systems using Lyapunov stability theory. Sufficient criteria for discrete-time singular systems with ROUs are derived based on linear matrix inequality (LMI) which can be easily solved by any LMI solvers. The main contribution of this paper is summarized as follows:

1. In order to predict the realistic behavior of the system, ROUs are considered in the system parameters.
2. The delay interval is split into $r$ number of equal subintervals and the LMI conditions are presented corresponding to each segment.
3. Convex reciprocal lemma is utilized to obtain less conservative results. Finally, numerical simulations are illustrated to show the effectiveness of the derived results.

Conventionally, in establishing Lyapunov functional, the delay interval $[\tau_m, \tau_M]$ is regarded as one segment, only one matrix variable is introduced corresponding to this segment. In this paper, a delay divisioning approach is utilized for the admissibility condition:

- The delay interval $[\tau_m, \tau_M]$ will be divided into $r$ equal-length subintervals, where $r$ is a positive integer. According to the division, $r$ number of matrix variables are introduced in the Lyapunov function.
- All the delay intervals are taken into account while formulating the Lyapunov function in order to provide more effective results.
- It will be proved that as $r$ increases, the admissibility condition becomes less conservative.
- One of the advantages of the proposed method is the enlargement of $r$ that can yield the maximum delay bound for admissibility.

The rest of the paper is organized as follows. In Section 2, the problem formulation and some preliminaries are given. The main results are provided in Section 3. In Section 4, numerical examples including Leontief model are demonstrated to show the effectiveness of the derived results. The conclusion is drawn in Section 5.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the $n$-dimensional Euclidean space and the set of all $n\times n$ real matrices respectively. The superscript $T$ and $(-1)$ denote the matrix transposition and matrix inverse respectively. The notation $X \geq Y$ (similarly, $X > Y$), where $X$ and $Y$ are symmetric matrices, means that $X-Y$ is positive semi-definite (similarly, positive definite). Matrices, if they are not explicitly stated, are assumed to have compatible dimensions. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $I$ is an identity matrix with appropriate dimension. The notation $*$ always denotes the symmetric block in a symmetric matrix. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation.

## 2. Problem description and preliminaries

Consider the following discrete-time singular system with time-varying delays:

$$\begin{cases} Ex(k+1)=(A+\alpha(k)\Delta A(k))x(k)+(A_d+\beta(k)\Delta A_d(k))x(k-\tau(k))+\sigma(k,x(k),x(k-\tau(k)))w(k) \\ x(s)=\phi(s), \quad s=-\tau_M,-\tau_M+1,\dots,0 \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector, $E$ is a singular matrix and rank $E=r<n$. $A$ and $B$ are constant matrices with appropriate dimensions. The real-valued matrices $\Delta A(k)$ and $\Delta A_d(k)$ represent the parameter uncertainties of the following structure:

$$\begin{bmatrix} \Delta A(k) & \Delta A_d(k) \end{bmatrix} = GF(k)\begin{bmatrix} H_1 & H_2 \end{bmatrix} \tag{2}$$

where $G, H_1, H_2$ are the constant matrices and $F(k)$ is the time-varying nonlinear function satisfying

$$F^T(k)F(k)\leq I. \tag{3}$$

Moreover, the uncertainties enter into the system parameters in a random sense. The stochastic variables $\alpha(k)$ and $\beta(k)$, which account for the phenomena of ROUs, are Bernoulli distributed white sequences taking values of 1 and 0 with

$$\text{Prob}\{\alpha(k)=1\}=\alpha, \quad \text{Prob}\{\alpha(k)=0\}=1-\alpha,$$
$$\text{Prob}\{\beta(k)=1\}=\beta, \quad \text{Prob}\{\beta(k)=0\}=1-\beta.$$

The time-varying delay $\tau(k)$ satisfies

$$0<\tau_m\leq\tau(k)\leq\tau_M$$

where $\tau_m$ and $\tau_M$ denote the lower and upper bounds of the time-delay. $\sigma(\cdot):\mathbb{N}^+\times\mathbb{R}^n\times\mathbb{R}^n\longrightarrow\mathbb{R}^n$ is a continuous function. $w(k)$ is a scalar Wiener process on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ with

$$\mathbb{E}\{w(k)\}=0, \quad \mathbb{E}\{w^2(k)\}=1, \quad \mathbb{E}\{w(i)w(j)\}=0 \ (i\neq j).$$

Using the structure of norm bounded uncertainties, system (1) can be rewritten as

$$\begin{cases} Ex(k+1)=Ax(k)+A_dx(k-\tau(k))+G\varrho(k)+\sigma(k,x(k),x(k-\tau(k)))w(k), \\ \varrho(k)=F(k)\varsigma(k), \\ \varsigma(k)=\alpha(k)H_1x(k)+\beta(k)H_2x(k-\tau(k)), \\ x(s)=\phi(s), \quad s=-\tau_M,-\tau_M+1,\dots,0. \end{cases} \tag{4}$$

**Assumption 1.** $\sigma(k,x(k),x(k-\tau(k)))$ is the continuous function satisfying $\sigma(k,0,0)=0$ and

$$\sigma^T(k,x(k),x(k-\tau(k)))\sigma(k,x(k),x(k-\tau(k)))\leq\rho_1x^T(k)x(k) +\rho_2x^T(k-\tau(k))x(k-\tau(k))$$

where $\rho_1$ and $\rho_2$ are two positive constant scalars.

Define the following scalar with respect to the variation range of time-delay $\tau(k)$:

$$\delta=\lceil\frac{\tau_M-\tau_m+1}{r}\rceil \tag{5}$$

where $\lceil x\rceil$ rounds $x$ towards the nearest integer to $x$. $\delta\geq1$. $r$ is a