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Delay-dependent finite-time boundedness of a class of Markovian switching neural networks with time-varying delays

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ABSTRACT

In this paper, a novel method is developed for delay-dependent finite-time boundedness of a class of Markovian switching neural networks with time-varying delays. New sufficient condition for stochastic boundness of Markovian jumping neural networks is presented and proved by an newly augmented stochastic Lyapunov–Krasovskii functional and novel activation function conditions, the state trajectory remains in a bounded region of the state space over a given finite-time interval. Finally, a numerical example is given to illustrate the efficiency and less conservative of the proposed method.

1. Introduction

During the past few decades, delayed neural networks are found successfully applied in the signal processing, image processing and pattern recognition problems [1–4]. However, these successful applications are major relay on the dynamic behaviors of delayed neural networks and some of these applications dependent on the stability of the equilibria of neural networks [5–7], and a large number of results on this topic has been investigated in the literature [8–10]. Among the recent advance in this area, delay-fraction method [11], free-weighting matrix technique [12], convex analysis approach [13] are the major effective ones. Up to now, the issue of stability for neural networks with delays has received an increasing attention.

On the other hand, stochastic systems have received much more attention since the stochastic modeling comes to play an major role in many real applications. It should be noted that, in most of the relevant literature, Markovian jump systems is an important class of stochastic systems. Recently, the Markovian process or Markovian chain can determines the switching among the different neural networks modes in [14,15]. Therefore, it is of great value both practically and theoretically to study of Markovian jump neural networks subject to time delay, and many relevant results has also been reported in the literature [16,17]. It should be pointed out that the neural networks with Markovian switching are always based on the Lyapunov method. On the other hand, it is very difficult to construct an appropriate Lyapunov functional, the stability criterion of many neural networks subject to Markovian switching obtained is often too conservative.

Nowadays, most of the existing literature are mostly concerned with the Lyapunov asymptotic stability, which is defined over an infinite-time interval. However, in practice, one is always interested in a bound of system trajectories over a fixed short time instead of Lyapunov asymptotic stability over an infinite-time interval, such as networked control systems [18,19]. Recently, some early results relating to finite-time stability can be found in [20–25]. In [23], the problem of finite-time H_∞ control for continuous-time Markovian jump systems has been investigated through the new Lyapunov functions. In [24], the issue of finite-time H_∞ control for discrete-time Markovian jump systems subject to average dwell time switching has been considered. In [25], by introducing a newly augmented Lyapunov–Krasovskii functional and considering the relationship between time-varying delays and their upper delay bounds, the problem of finite-time filtering for switched linear systems with a mode-dependent average dwell time. With a new integral inequality, the finite-time H_∞ control for a class of nonlinear system with time-varying delay, which can be represented by the Takagi–Sugeno fuzzy system in [26].

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To the best of authors' knowledge, the issue of delay-dependent finite-time boundedness about neural networks with time-varying delays and Markovian switching has not been fully tackled except for [20]. It should be mentioned that, in [20], there still exists some conservative when discuss the stability criteria of neural networks. There still exists room for further improvement due to the fact that some useful terms are ignored in the stochastic Lyapunov–Krasovskii functional in [20]. It is natural to look for an alternative view to reduce the conservatism of stability criteria. This motivates our research on this topic.

In this paper, the issue for delay-dependent finite-time boundedness of neural networks with time-varying delays and Markovian switching is investigated. In order to solve the term like $\int_{t-\tau}^{t}\langle \dot{z}(s), Q\dot{z}(s)\rangle\, ds$, a new processing way is proposed in Lemma 2.3. Making full use of nonlinear parameters and time-varying delays, less conservative stability condition for finite-time boundedness is derived. At last, a numerical example is given to illustrate the efficiency and less conservative of the proposed method.

Notations: Throughout of paper, letting $P>0(P\geq 0, P<0, P\leq 0)$ denote a symmetric positive definite matrix P (positive-semi definite, negative definite and negative-semi definite). For any symmetric matrix P, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ respectively denotes the corresponding maximum and minimum eigenvalues of matrix P. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space and $\mathcal{R}^{n\times m}$ represents the set of all $n\times\ m$ real matrices. The shorthand col$\{Z_1, Z_2, \ldots, Z_n\}$ denotes a column matrix with the matrices $Z_1, Z_2, \ldots, Z_n$. The identity matrix of order n is denoted as I_n. $*$ represents the elements below the main diagonal of a symmetric matrix. The superscripts $\top$ and -1 represents for matrix transposition and matrix inverse, respectively.

2. Preliminaries

Given a probability space (Ω, F, P) where Ω, F and P respectively represents the sample space, the algebra of events and the probability measure. In this paper, we consider the following n-neuron Markovian jumping neural network over the space (Ω, F, P) described by

$$\begin{cases} \dot{x}(t) = -A_{r_t}x(t) + B_{r_t}f(x(t)) + C_{r_t}f(x(t-\tau_{r_t}(t))) + J \\ x(t) = \phi(t),\ t\in[-\tau, 0). \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^\top$ represents the neural state vector of the system, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^\top$ denotes the nonlinear activation function with the given initial condition $f(0)=0$; $A_{r_t} = \text{diag}\{a_1(r_t), a_2(r_t), \ldots, a_n(r_t)\}$ describes the rate with every neuron will reset its potential to the resting state in isolation; $B_{r_t} = [b_{ij}(r_t)]_{n\times n}$ and $C_{r_t} = [c_{ij}(r_t)]_{n\times n}$ are respectively denotes the connection weight matrix and the delayed connection weight matrix; $J = [J_1, J_2, \ldots, J_n]^\top$ represents a constant external input vector. $\tau_{r_t}(t)$ is the time-varying delay which satisfies

$$0\leq \tau_{r_t}(t)\leq \tau_{r_t}, \tag{2}$$

$$h_{0r_t}\leq \dot{\tau}_{r_t}(t)\leq h_{Mr_t}. \tag{3}$$

where τ_{r_t}, h_{0r_t} and h_{Mr_t} are constant scalars, and $\tau = \max_{r_t}\{\tau_{r_t}\}$, $h_0 = \min_{r_t}\{h_{0r_t}\}$, $h_M = \max_{r_t}\{h_{Mr_t}\}$.

There exists a parameter $1\geq\chi\geq 0$ such that $\dot{\tau}_{r_t}(t)$ can be expressed as a convex combinations of the vertices

$$\dot{\tau}_{r_t}(t) = \chi h_{0r_t} + (1-\chi)h_{Mr_t}. \tag{4}$$

Remark 2.1. It should be noted that if the stability condition is dependent on $\dot{\tau}_{r_t}(t)$, it only needs to check the vertex values of $\tau_{r_t}(t)$ instead of all values of $\dot{\tau}_{r_t}(t)$. This technique is employed to reduce the conservatism of stability criteria for Markovian jump systems with time-varying delays.

Letting the random form process $\{r_t, t\geq 0\}$ being the Markov stochastic process and taking values on a finite set $\mathcal{N} = \{1, 2, \ldots, N\}$ with transition rate matrix $\Omega = \{\pi_{ij}\}$, $i, j\in\mathcal{N}$, namely for $r_t = i$, $r_{t+1} = j$, one has

$$\Pr(r_{t+h} = j|\, r_t = i) = \begin{cases} \pi_{ij}h + o(h) & \text{if } j\neq i \\ 1+\pi_{ii}h + o(h) & \text{if } j = i \end{cases}$$

where $h>0$, $\lim_{h\to 0}(o(h)/h) = 0$ and $\pi_{ij}\geq 0$ $(i, j\in\mathcal{N}, j\neq i)$ denotes switching rate from mode i to mode j. $\forall i\in\mathcal{N}$, $\pi_{ii} = -\sum_{j=1, j\neq i}\pi_{ij}$.

Setting $\mathcal{N}$ contains N in the mode of system (1) and for $r_t = i\in\mathcal{N}$, the system matrices of the ith mode are always denoted by A_i, B_i and C_i, all the matrices are considered to be known with appropriate dimensions.

Assumption 2.1. The neuron state-based nonlinear function $f(x(t))$ in system (1) is bounded and satisfies

$$\gamma_s^- \leq \frac{f_s(\varsigma_1) - f_s(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq \gamma_s^+,\ \varsigma_1\neq\varsigma_2,\ s = 1, 2, \ldots, n \tag{5}$$

for all ς_1, $\varsigma_2\in\mathcal{R}$, with γ_s^- and γ_s^+ are known real constants with $s = 1, 2, \ldots, n$.

Remark 2.2. It is seen from Assumption 2.1 that γ_i^- and γ_i^+ can be allowed to be negative, positive, or zero. As mentioned in [10], Assumption 2.1 describes the monotone nondecreasing activation when $\gamma_i^- = 0$ and $\gamma_i^+ > 0$. Moreover, monotone increasing activation functions can be described when $0<\gamma_i^- < \gamma_i^+$.

Remark 2.3. The lower bound of activation function was settled to zero in [1,7] and the stability criteria obtained in [1,7] is conservatism. However, the activation function in (5) is more general.

Noting that with the Brouwers fixed-point technique, it can be easily proved that Markovian jump systems (1) has one equilibrium point. Assuming that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^\top$ is the equilibrium point of (1) and using the transformation $z(\cdot) = x(\cdot) - x^*$, system (1) can be converted as follows:

$$\dot{z}(t) = -A_{r_t}z(t) + B_{r_t}g(z(t)) + C_{r_t}g(z(t-\tau_{r_t}(t))), \tag{6}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^\top$, $g(z(\cdot)) = [g_1(z_1(x(t)), g_2(x(t))), \ldots, g_n(x(t))]^\top$ and $g_i(z_i(z_i(\cdot))) = f_i(z_i(\cdot) + x_i^*) - f_i(x_i^*)$, $i = 1, 2, \ldots, n$. According to Assumption 1, one can obtain that

$$\gamma_i^- \leq \frac{g_i(\varsigma_1) - g_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq \gamma_i^+,\quad g_i(0) = 0,\ i = 1, 2, \ldots, n \tag{7}$$

It should be pointed out that, for given $0<\varrho<1$, in order to improve the stability condition of feasible region, the term $0\leq\tau_{r_t}(t)\leq\tau_{r_t}$ is always divided into $0\leq\tau_{r_t}(t)\leq\varrho(\tau_{r_t}/2)$ and $\varrho(\tau_{r_t}/2)\leq\tau_{r_t}(t)\leq\tau_{r_t}$. On the other hand, the activation functions in (6) is also satisfied the condition (7). Similar to the above division, the bounding of activation function (7) can be divided as follows:

$$\gamma_i^- \leq \frac{g_i(\varsigma_1) - g_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq \varrho\delta, \tag{8}$$

$$\varrho\delta \leq \frac{g_i(\varsigma_1) - g_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq \gamma_i^+ \tag{9}$$

where $\delta = (\gamma_i^- + \gamma_i^+)/2$.

Instead of using the delay-partitioning approach, this technique mainly improves the feasible region of stability criterion. In this paper, the terms $(\varsigma_1, \varsigma_2)$ in (8) and (9) are replaced by terms $(z(t), z(t-\kappa\tau_{r_t}(t)))$, $(z(t-\kappa\tau_{r_t}(t)), z(t-\tau))$ and $(z(t-\tau_{r_t}(t)), z(t-\tau))$ at each subintervals. Furthermore, the cross terms among the states $z(t)$, $z(t-\kappa\tau_{r_t}(t))$, $z(t-\tau_{r_t}(t))$, $z(t-\tau)$, $f(z(t))$, $f(z(t-\kappa\tau_{r_t}(t)))$, $f(z(t-\tau_{r_t}(t)))$