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Robust pole assignment using velocity–acceleration feedback for second-order dynamical systems with singular mass matrix

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ABSTRACT

In this paper the robust pole assignment problem using combined velocity and acceleration feedback for second-order linear systems with singular mass matrix is illustrated. This is promising for better applicability in several practical applications where the acceleration signals are easier to obtain than the proportional ones. First, the explicit parametric expressions of both the feedback gain controller and the eigenvector matrix are derived. The parametric solution involves manipulations only on the original second-order model. The available degrees of freedom offered by the velocity–acceleration feedback in selecting the associated eigenvectors are utilized to improve robustness of the closed-loop system. Straight-forward computational algorithms are introduced to demonstrate the effectiveness of the proposed approach. These algorithms are applicable for a dynamical system with mass matrices that can be either singular or nonsingular. Numerical examples are provided to illustrate the application of the proposed procedure.

1. Introduction

This work is an extension of the recently published papers [1,2]. In these papers the dynamical system is described by matrix second-order, time-invariant, differential equations of the form

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{D}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{C}\boldsymbol{u}(t),$$
$$\boldsymbol{x}(0)=\boldsymbol{x}_0,\quad \dot{\boldsymbol{x}}(0)=\dot{\boldsymbol{x}}_0 \tag{1.1}$$

by velocity–acceleration feedback control

$$\boldsymbol{u}(t)=-\boldsymbol{F}_v\dot{\boldsymbol{x}}(t)-\boldsymbol{F}_a\ddot{\boldsymbol{x}}(t) \tag{1.2}$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the generalized coordinate vector and $\boldsymbol{u}(t)\in\mathbb{R}^r$ is the vector of applied forcing. The real matrices $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}\in\mathbb{R}^{n\times n}$ and $\boldsymbol{C}\in\mathbb{R}^{n\times r}$ are, respectively, the mass, damping, stiffness and control matrices. Moreover, $\boldsymbol{F}_v$, $\boldsymbol{F}_a\in\mathbb{R}^{r\times n}$ are, respectively, velocity and acceleration gain matrices, which assign the prescribed closed-loop eigenvalues and eigenvectors. However, in [1,2] the mass matrix $\boldsymbol{M}$ is regular while in the present study the mass matrix $\boldsymbol{M}$ is singular, $\text{rank}(\boldsymbol{M})=q<n$, $q>0$. Then, the system is called singular, generalized, or descriptor second-order system; see [3–12]. One inherent characteristic of descriptor second-order linear systems is their impulsive natural response which is generated by infinite eigenvalues. In fact, impulses may cause degradation in performance, damage components, or even destroy the system. Therefore, eliminating the impulsive behavior of a descriptor system via certain feedback control is an important problem in descriptor systems theory. In this case, infinite eigenvalues require a special treatment.

By the substitution of Eq. (1.2) into Eq. (1.1), we can obtain the closed-loop system

$$(\boldsymbol{M}+\boldsymbol{C}\boldsymbol{F}_a)\ddot{\boldsymbol{x}}(t)+(\boldsymbol{D}+\boldsymbol{C}\boldsymbol{F}_v)\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{0}. \tag{1.3}$$

The effect of the closed-loop control is therefore to modify the mass and damping parameters so that the unstable system becomes stable. Thus, one of the advantages of the combined velocity and acceleration feedback is permitting the treatment of second-order systems involving singular mass matrices.

Second-order systems with singular mass matrix arise naturally in mechanical multi-body systems and a variety of other practical applications; see [3–12]. In 1982, the second order generalized systems are applied to power systems by Campbell and Rose [3]. Moreover, the generalized systems were used in the analysis and modeling of flexible beams [4]. The use of zero-mass points for example to denote a connection between two springs or two dampers may be the cause of singularities in the resulting models [5]. The treatment of some forces like reaction forces or friction forces as unknowns of the problem may also introduce singularities in the whole model [5]. The mathematical strategies permitting the

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treatment of second order unilateral systems involving singular mass, damping, and stiffness matrices are discussed in [6]. The explicit equation of motion for constrained mechanical systems with singular mass matrices has been derived in [7]. Especially in modeling complex multibody systems, it is useful to use redundant coordinates, and in such situations singular mass matrices can arise when describing the unconstrained system [7]. In addition, when a mass matrix $\boldsymbol{M}$ is invertible but contains very small terms such that $\boldsymbol{M}$ is ill-conditioned, it is common practice to set these terms to zero, rendering $\boldsymbol{M}$ singular [11]. The controllability and observability conditions for descriptor second-order linear systems have been analyzed in [8]. The eigenvalue assignment with minimum sensitivity for descriptor second-order systems via proportional-derivative state feedback is proposed [9]. The output regulation for matrix second-order descriptor systems using output feedback measurement is presented [10]. Moreover, a procedure for decoupling a second-order linear system with a singular mass matrix is developed [11]. Recently, some new results on the eigenstructure assignment for the second-order system with a singular mass matrix are developed in [12]. The first-order descriptor systems (singular systems or generalized systems) have been of great interest in the specialized literature because they have many practical applications in robotics, mechanical systems, electrical circuit networks, economics and other areas; see [13–15] and the references therein. Up to date, no results on robust pole assignment control problem for descriptor second-order linear systems are available in the literature using combined acceleration and velocity feedback, this problem is still open.

The model of second-order linear systems has found wide applications in many engineering and scientific fields, including control of flexible space structures, robotics control, control of mechanical multi-body systems, earthquake engineering, vibration control in structural dynamics, electrical circuit simulation and microelectromechanical systems (MEMS). The problem of maintaining the stability of the second-order system using combined proportional and derivative feedback has been an active area of research; see [16–28] and the references therein. There have been various approaches concerning robust pole assignment [9,16–19], robust eigenstructure assignment [20], robust partial pole assignment [21], partial pole assignment [22], pole assignment [23,24], eigenstructure assignment (ESA) [25–27], and optimal control [28] for second-order systems. In a previous study for second-order systems, the measured system responses were assumed to be displacements and velocities. However, these types of system responses, especially for large flexible structures, are not as easily and accurately obtainable as accelerations, which have been commonly obtained through the use of accelerometers. Accelerometer is a favorable sensor to measure the dynamic structural responses from the viewpoint of measurement. Acceleration is often easier to measure than displacement or velocity, particularly when the structure is stiff [29]. From measured accelerations, it is possible to reconstruct velocities with reasonable accuracy. Therefore, the available signals for feedback are accelerations and velocities. One necessary condition for a control strategy to be implementable is that it must utilize the available measured responses to determine the control action. Recently, the full eigenstructure assignment and the partial quadratic eigenvalue assignment for second-order linear systems using combined acceleration and velocity feedback are proposed [12,30]. Moreover, the partial eigenstructure assignment for undamped vibration systems using combined acceleration and displacement feedback is introduced in [31]. Consequently, the velocity and acceleration variables can be utilized and new design techniques for controlling such practical systems should be developed.

The derivative feedback control in first-order linear systems has been considered by many researchers; see [32–36]. Accordingly, derivative feedback methodology that utilizes the combined acceleration and velocity variables has been recently attracted the attention of many authors to all aspects of control, including robust pole assignment, eigenstructure assignment, optimal control, and stabilization [32–36]. The researchers have recognized the importance of using derivative feedback on diverse practical engineering fields, such as vibration suppression in mechanical systems and flexible structures where the accelerometer is the only sensor. Moreover, the problem of state-derivative feedback has been investigated within the treatment of a generalized class of first-order singular linear systems [15]. Consequently, the derivative feedback control has been applied to design numerous kinds of practical systems.

The main contribution of the present research work is to introduce a novel procedure for robust controller design for the second-order linear system with singular mass matrix using combined velocity and acceleration variables. First, the explicit necessary and sufficient conditions that ensure solvability for the proposed problem are introduced. Second, the parametric expressions for both the feedback gain controllers and the eigenvector matrices are presented which describe the available degrees of freedom offered by the velocity–acceleration feedback. Based on these parametric expressions, a performance index is proposed to utilize the available degrees of freedom to obtain the robust solution. So, these freedoms are utilized to improve robustness of the closed-loop system. Consequently, the robust pole assignment problem is considered and an effective method that finds the robust gain controller is obtained. Two illustrative examples are given to show that our results are effective. Finally, conclusions are made.

2. Problem formulation

For the second-order linear system (1.1), the corresponding quadratic polynomial pencil is

$$\boldsymbol{P}_o(\lambda) = \lambda^2\boldsymbol{M} + \lambda\boldsymbol{D} + \boldsymbol{K}, \quad \boldsymbol{P}_o(\lambda) \in \mathbb{R}^{n\times n}[\lambda],$$

whose determinant is the characteristic polynomial of system (1.1).

Definition 2.1. Given second-order system (1.1), the matrix pencil $\boldsymbol{P}_o(\lambda)$ is called regular if and only if $\det(\boldsymbol{P}_o(\lambda))$ is not identically zero. In any other case, the pencil will be called singular.

The zeroes of $\det(\boldsymbol{P}_o(\lambda))$ (finite as well as infinite) are known as the characteristic frequencies of the system and play an important role in system stability. For descriptor second-order linear systems, at least one eigenvalue is infinite. The characteristic frequencies of the system can be obtained by solving

$$\det(\boldsymbol{P}_o(\lambda)) = \alpha_{2n}\lambda^{2n} + \alpha_{2n-1}\lambda^{2n-1} + \ldots + \alpha_1\lambda + \alpha_0 = 0,$$

where $\alpha_{2n} = \det(\boldsymbol{M})$ and $\alpha_0 = \det(\boldsymbol{K})$. If matrix $\boldsymbol{M}$ is nonsingular then the polynomial $\boldsymbol{P}_o(\lambda)$ of degree $2n$ and all the eigenvalues of $\boldsymbol{P}_o(\lambda)$ are finite. Otherwise, if $\boldsymbol{M}$ is singular then $\boldsymbol{P}_o(\lambda)$ is said to have infinite eigenvalues which may be identified as the zero eigenvalues of the reverse or dual polynomial $\lambda^2\boldsymbol{P}_o(1/\lambda) = \lambda^2\boldsymbol{K} + \lambda\boldsymbol{D} + \boldsymbol{M}$. Let n_f and n_∞ denote the finite eigenvalues counting algebraic multiplicities and the eigenvalue at infinity of algebraic multiplicity, respectively, then $n_f + n_\infty = 2n$. The number of finite eigenvalues is given precisely by $n_f = \deg(\det(\boldsymbol{P}_o(\lambda)))$.

The objective of feedback is to eliminate the impulsive behavior which is generated by infinite eigenvalues for a descriptor system. The problem is to find $\boldsymbol{F}_v$ and $\boldsymbol{F}_a$ such that the eigenvalues and eigenvectors of the associated closed-loop quadratic pencil

$$\boldsymbol{P}_c(\lambda) = \lambda^2(\boldsymbol{M} + \boldsymbol{C}\boldsymbol{F}_a) + \lambda(\boldsymbol{D} + \boldsymbol{C}\boldsymbol{F}_v) + \boldsymbol{K} \tag{2.1}$$

can be altered as required to ensure and improve the stability of the closed-loop system. It is known that the behavior of the closed-loop system (1.3) is governed by the eigenstructure of its associated quadratic polynomial $\boldsymbol{P}_c(\lambda) \in \mathbb{R}^{n\times n}[\lambda]$. The closed-loop