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Pinning synchronization of fractional-order complex networks with Lipschitz-type nonlinear dynamics

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ABSTRACT

This paper deals with pinning synchronization problem of fractional-order complex networks with Lipschitz-type nonlinear nodes and directed communication topology. We first reformulate the problem as a global asymptotic stability problem by describing network evolution in terms of error dynamics. Then, a novel frequency domain approach is developed by using Laplace transform, algebraic graph theory and generalized Gronwall inequality. We show that pinning synchronization can be ensured if the extended network topology contains a spanning tree and the coupling strength is large enough. Furthermore, we provide an easily testable criterion for global pinning synchronization depending on fractional-order, network topology, oscillator dynamics and state feedback. Numerical simulations are provided to illustrate the effectiveness of the theoretical analysis.

1. Introduction

Concurrent technological advancements have been generating large datasets from diverse complex systems, such as biological and chemical systems, neural networks, social species, the Internet and the World Wide Web, just to mention a few. In the process of characterizing the large datasets, a multidisciplinary methodology called complex network is emerging as a powerful tool to the study of complex systems [1–3]. Complex networks consist of nodes (or vertices) representing network elements and links (or edges) representing interaction pathways between the elements. The study of complex networks has revealed many interesting and common phenomena of different real-life networks: the small-world property [4], the scale-free feature [5], the overlapping community structure [6,7], the hierarchical structure [8], among others. These features cannot be well described by the completely random graphs and completely regular graphs, thus clearly distinguishing them from complex networks.

An active field of complex networks is the synchronization phenomenon occurring on them, which constitutes one of the most paradigmatic examples of emerging collective behavior in nature and society [9]. In the past few years, much research effort has been dedicated to unveiling the influence of interaction topologies on the onset of network synchronization and its stability [10–19]. However, the above important findings are mainly focused on synchronization of complex networks by their intrinsic topological structure (i.e., without requiring any external controllers). Actually, for some real-world complex networks, it is more desirable to synchronize all the nodes to a desired state (e.g., an equilibrium point, a periodic orbit or a chaotic orbit). In this case, appropriate controllers should be added to drive them to the desired state. Combining the fact that real-world networks normally have a large number of nodes, controlling a complex network by adding controllers to all its nodes is neither practical nor economical. Thus, an effective method to solve this problem is adding controllers to just a limited subset of the network nodes. This approach is known as pinning control, which was first proposed to control chaos in spatiotemporal systems [20–22] and proved to be a viable strategy to tame complex networks with different topological structures [23–31].

Despite the advances in the study of pinning synchronization, former works are devoted entirely to integer-order complex networks, in which node dynamics are described by ordinary differential equations. It has been shown that integer-order complex networks is inefficient to model real-life complex systems with memory and hereditary properties [32–37]. In this regard, fractional-order complex networks (FCNs), in which the node dynamics are described by fractional differential equations, provide an excellent tool to model memory and hereditary properties of such systems [38]. In recent years, several theoretical studies have been carried out in attempts to explain the synchronization of FCNs [39–46]. For example, by designing a nonlinear coupling

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scheme, we proposed a general FCN [40] whose synchronous behavior is different from its constituent elements due to the nondiffusive coupling. We also explored the robustness of outer synchronization between unidirectionally coupled FCNs with respect to uncertain system parameters [41]. The work of Wu et al. [43] has shown that the scenario of generalized synchronization can also be realized in a FCN with nonidentical nodes. However, in previous FCNs, the synchronization behavior is produced either by relying on their intrinsic topological structure or by applying the control action to all the network elements. Although there are few works on synchronization problem of FCNs via pinning control [47–49], they all employ Lyapunov-based method which is difficult to construct for fractional-order systems. Theoretically, the pinning synchronization of FCNs with general topological structure is more challenging than that for integer-order complex networks. This topic still remains to be an open problem and needs to be more deeply solved.

Motivated by the above discussions, this paper presents a further investigation of the subject, in which we want to stabilize the FCNs with Lipschitz-type nonlinear nodes onto some desired homogenous stationary states of the isolated nodes by pinning a small fraction of network units. The contribution of the paper is twofold. First, we present a new approach to analyze the pinning synchronization problem of FCNs by using Laplace transform, algebraic graph theory and the generalized Gronwall inequality. Second, an easily testable sufficient condition for global pinning synchronization is established in terms of fractional-order, network topology, oscillator dynamics, and linear feedback. We show that the pinning synchronization in FCNs can be achieved if the extended network topology contains a directed spanning tree and the coupling strength is large enough.

The rest of this paper is organized as follows. In Section 2, we review a few concepts of fractional-order derivatives and algebraic graph theory, and formally define the pinning synchronization problem for the FCNs. In Section 3, the global synchronization stability via pinning control is analyzed and the main results are presented. Numerical simulations that illustrate the effectiveness of theoretical results are given in Section 4. Finally, in Section 5 conclusions are given based on the results obtained in this work.

Our notation throughout is standard. $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{Z}^+$ refer to the sets of real numbers, complex numbers and nonnegative integers, respectively. The n-dimensional Euclidean space and the set of $n\times m$ real matrices are indicated with $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, respectively. $\mathrm{diag}\{z_1, z_2, \ldots, z_m\}$ denotes the diagonal matrix with diagonal entries z_1 to z_m. $\|\cdot\|$ refers to the Euclidean norm in $\mathbb{R}^n$ or corresponding induced matrix norm in $\mathbb{R}^{n\times n}$.

2. Problem formulation and preliminaries

2.1. Fractional-order derivatives

Fractional-order derivative is a generalization of the integer-order ones. There are different definitions for fractional derivatives. The frequently used definitions are the Grünwald–Letnikov, Riemann–Liouville, and Caputo definitions [34,38]. Because the Caputo definition is suitable to be treated by the Laplace transform technique with clear physical interpretations, it is more preferred in modeling practical problems. Therefore, in the rest of the paper, we use $D^\alpha g(t)$ to denote the Caputo fractional derivative of order α of function $g(t)$

$$D^\alpha g(t) = \frac{d^\alpha g(t)}{dt^\alpha} = \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{g^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau,$$

where m is an integer satisfying $m-1<\alpha\le m$ and $\Gamma(\cdot)$ is the Gamma function $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}\,dt$. This paper mainly focuses on the synchronization problem of FCNs with fractional order $1\le\alpha<2$.

As the role of the exponential function e^z in the theory of integer-order differential equations, the Mittag–Leffler function plays a very important role in the theory of FDEs. A two-parameter function of the Mittag–Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},$$

where $\alpha>0$, $\beta>0$, and $z\in\mathbb{C}$.

2.2. Model description

Consider a FCN consisting of N coupled nodes, in which each node is an n-dimensional fractional-order differential system. The entire FCN is described by

$$D^\alpha x_i(t) = f(x_i(t)) - k\sum_{j=1}^{N} l_{ij}x_j(t), \quad i=1,2,\ldots,N, \tag{1}$$

where $1\le\alpha<2$ is the fractional order, $x_i(t)=(x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T\in\mathbb{R}^n$ is the state vector of the ith node, $f(\cdot)\in\mathbb{R}^n$ is a continuous map describing the node individual dynamics, scalar $k>0$ is the coupling strength, and $\mathcal{L}=(l_{ij})\in\mathbb{R}^{N\times N}$ is the coupling configuration matrix determining the topological structure of the network. Its entries l_{ij} are defined as follows: if there is a link from node j to node i, then $l_{ij}<0$, otherwise $l_{ij}=0$ $(i\ne j)$; and the diagonal elements l_{ii} of matrix $\mathcal{L}$ are defined by $l_{ii}=-\sum_{j=1,j\ne i}^{N} l_{ij}$, which ensures the diffusion that $\sum_{j=1}^{N} l_{ij}=0$.

For the FCN (1), if each node is regarded as a vertex and each communication link is regarded as an edge, then its coupling topology can be conveniently described by a simple graph (for more details on graph theory, the interested readers please refer to some textbooks [50]). Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{W})$ be a weighted directed graph (digraph) of order N, where $\mathcal{V}=\{1,\ldots,N\}$ denotes the vertex set, $\mathcal{E}=\{e(i,j)\}$ denotes the directed edge set by that $e(i,j)\in\mathcal{E}$ if and only if there exists a directed edge from vertex j to vertex i, and a weighted adjacency matrix $\mathcal{W}=[w_{ij}]\in\mathbb{R}^{N\times N}$. The element w_{ij} is decided by the edge $e(i,j)$, i.e., $e(i,j)\in\mathcal{E}\Leftrightarrow w_{ij}>0$, otherwise $w_{ij}=0$. The neighbor set of vertex i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}\,|\,(i,j)\in\mathcal{E}\}$. The Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ of the digraph $\mathcal{G}$ is defined by $\mathcal{L}=\mathcal{D}-\mathcal{W}$, where $\mathcal{D}=\mathrm{diag}\{d_1,d_2,\ldots,d_N\}$ and $d_i=\sum_{j=1,j\ne i}^{N} w_{ij}$ is the in-degree of vertex i. A directed path from vertex j to vertex i is a sequence of edges $e(i,i_1)$, $e(i_1,i_2),\ldots,e(i_m,j)$ with distinct nodes $i_k, i=1,2,\ldots,m$. We say that a digraph $\mathcal{G}$ has *a spanning tree* if there exists at least one vertex called *root* which has directed paths to all the other vertices. Obviously, the coupling configuration matrix $\mathcal{L}$ of the FCN (1) denotes the Laplacian of its corresponding digraph $\mathcal{G}$, and by the way we can obtain the corresponding weighted adjacency matrix $\mathcal{W}$.

The objective of this paper is to design some appropriate controllers for the FCN (1) such that its solutions globally synchronize with the solution of the uncoupled node satisfying

$$D^\alpha x_0(t) = f(x_0(t)), \tag{2}$$

in the sense that $\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0$ $(i=1,2,\ldots,N)$, where $x_0(t)$ can be an equilibrium point, a periodic orbit, an aperiodic orbit, or even a chaotic trajectory in the phase space. For this purpose, we apply the pinning control strategy with local linear feedback controllers to a small fraction $\delta(0<\delta\le 1)$ of the total nodes in the FCN (1). Suppose that nodes $i_1,i_2,\ldots,i_l$ in the FCN (1) are selected to be pinned, where $l=\lfloor\delta N\rfloor$ is the smaller but nearest integer to δN. Without loss of generality, assume that the first l nodes are selected to be pinned. Otherwise, we can rearrange the order of the nodes. Therefore, the pinning controlled form of the