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Stability and stabilization studies for a class of switched nonlinear systems via vector norms approach

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A B S T R A C T

This paper is concerned with the problems of stability analysis and stabilization with a state feedback controller through pole placement for a class of both continuous and discrete-time switched nonlinear systems. These systems are modeled by differential or difference equations. Then, a transformation under the arrow form is employed. Note that, the main contribution in this work is twofold: firstly, based on the construction of an appropriated common Lyapunov function, as well the use of the vector norms notion, the recourse to the Kotelyanski lemma, the $M-matrix$ proprieties, the aggregation techniques and the application of the Borne–Gentina criterion, new sufficient stability conditions under arbitrary switching for the autonomous system are deduced. Secondly, this result is extended for designing a state feedback controller by using pole assignment control, which guarantee that the corresponding closed-loop system is globally asymptotically stable under arbitrary switching.

The main novelties features of these obtained results are the explicitness and the simplicity in their application. Moreover, they allow us to avoid the search of a common Lyapunov function which is a difficult matter. Finally, as validation to stabilize a shunt DC motor under variable mechanical loads is performed to demonstrate the effectiveness of the proposed results.

1. Introduction

Switched systems are a particular kind of hybrid dynamical systems that consist of a set of subsystems that are modeled by differential of difference equations and a switching signal, which defines a specific subsystem being activated during a certain interval. Switched systems have gained a great deal of attention in recent years, because many real-world systems such as mechanical systems, automotive industry, aircraft and air traffic control, chemical and electrical engineering can be modeled as switched systems [1–5].

The last decade has witnessed increasing research activities in the study of controllability, stability, stabilization of switched systems. Among these research topics, stability analysis and stabilization have attracted most of the attention [5–17]. Therefore, stability under arbitrary switching is a fundamental in the design and analysis of switched systems. In fact, this problem has been difficult and essential in researches. Within this framework, it is necessary that all the subsystems be asymptotically stable. However, this above mentioned condition is not sufficient to guarantee the stability under arbitrary switching. In this context, the existence of a common Lyapunov function for all subsystems was proved to be sufficient condition for switched system to be asymptotically stable under arbitrary switching.

Up to now, despite some attempts which are presented for the general constructions of a common Lyapunov function for switched nonlinear systems by using the Lyapunov converse theorems [11,12]. Also, in [13] by introducing commute matrix condition, which is relaxed to some nilpotent Lie algebras in [14]. But, the problem of the existence of a common Lyapunov function remainders very difficult even for a family of linear stationary systems [5,7].

Frequently, to avoid the problem of the existence of a common Lyapunov function, we are required to seek conditions that guarantee the stability of the switched systems under admissible switched law. Although many efficient approaches and important results have been proposed for this alternative such as the multiple Lyapunov function approach [15] and average dwell time method [16,17], stability under arbitrary switching which is considered in this work remains most preferable for practical systems. Indeed, it offers great flexibility and it allows us to achieve other performances for designing a control law along stability maintained.

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Thus far, despite, many interested and significant results on stability analysis and stabilization problems for switched systems have been established, unfortunately due to the complexity of switched nonlinear systems the available results for those systems are very limited [21].

With motivation from these mentioned shortcomings for the existed results, as well in the sense of various methods that can be employed, this paper attempts to establish novel stability analysis and a new state feedback controller design through pole placement for a class of both continuous and discrete-time switched nonlinear systems. Indeed, by constructing an appropriated common Lyapunov function [18–20], as well by using the vector norms notion [18–37] and also by resorting to the Kotelyanski lemma [38], the $M-$matrix proprieties [39,40], the aggregation techniques [34] and the application of the Borne–Gentina criterion [34,36], new sufficient stability conditions under arbitrary switching for the autonomous system are deduced. Then, these results are extended for designing a state feedback controller by using pole assignment control, which guarantee that the corresponding closed-loop system is globally asymptotically stable under arbitrary switching.

Within the frame of studying the stability analysis and the stabilization, the above mentioned approach has already been introduced in our previous work for continuous-time systems [23,25] and in [18,19,22,24] for discrete-time systems in a field related to the study of convergence.

With this method and from a theoretical point of view, we promise it an alternative way to the search for a common Lyapunov function. As well this approach can be very effective in dealing with a class of more general nonlinear systems.

Note that, the main contribution in this work is twofold: first, due to the conservatism of the methods for stability analysis which are based on the common Lyapunov function. This proposed method can guarantee the stability under arbitrary switching and allows us to avoid the search of a common Lyapunov function. Second, a new stabilization under arbitrary switching with state feedback controller through pole placement is proposed. Finally, a validation of the obtained results is shown to stabilize a shunt DC motor under variable mechanical loads.

The body of this paper is organized as follows: Section 2 presents the problem formulation and the main results for the studied continuous-time switched nonlinear systems. In Section 3, we present the research problem formulations and the main results of the studied discrete-time switched nonlinear systems. In Section 4, the effectiveness of the proposed design results is demonstrated by two examples. Section 5 concludes the paper.

Notations. Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. I_n is an identity matrix with appropriate dimension. Let $\Re^n$ denoted an n dimensional linear vector space over the reals and $\|.\|$ stands for the Euclidean norm of vectors. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$, we define the scalar product of the vector u and v as: $\langle u,v\rangle=\sum_{i=1}^{n}u_iv_i$. $\lambda(M)$ denote the set of eigenvalues of matrix M, M^T its transpose and M^{-1} its inverse if $M=(m_{i,j})_{1\le i,j\le n}$. We denote $M^*=\left(m^*_{i,j}\right)_{1\le i,j\le n}$ with $m^*_{i,j}=m_{i,j}$ if $i=j$ and $m^*_{i,j}=|m_{i,j}|$ if $i\ne j$ and $|M|=|m_{i,j}|$, $\forall i,j$. $\underline{N}=\{1,2,\dots,N\}$.

2. Continuous-time switched nonlinear systems

2.1. Problem statement and preliminaries

2.1.1. Statement of the problem

Consider the following switched nonlinear system:

$$\dot{x}(t)=A_{\sigma(t)}(.)x(t)+B_{\sigma(t)}(.)u(t) \tag{1}$$

where $x(t)\in\Re^n$ is the state, $u(t)\in\Re$ is the control input, let $u(t)=-k_{\sigma(t)}x(t)$ be the state feedback controller, $A_{\sigma(t)}(.)$, $B_{\sigma(t)}(.)$ are matrices with appropriate dimensions, and $\sigma(t):\Re^+\to\underline{N}=\{1,2,\dots,N\}$ is the switching signal assumed to be available in real time. Therefore, the continuous-time switched nonlinear system is composed of N continuous-time subsystems which are expressed as:

$$\dot{x}(t)=A_i(.)x(t)+B_i(.)u(t) i\in\underline{N} \tag{2}$$

where $A_i(.)$ $i\in\underline{N}$ and $B_i(.)$ $i\in\underline{N}$ are matrices that have nonlinear elements with appropriate dimensions.

Define the following functions $\nu_i(t)$, $\forall i\in\underline{N}$, that will be used to represent our system:

$$\nu_i(t)=\begin{cases}1 & \text{if } \sigma(t)=i,\\ 0 & \text{otherwise,}\end{cases} i\in\underline{N}; \tag{3}$$

where $\sigma(t)$ is defined in (1). The switched system (1) can be rewritten as follows:

$$\dot{x}(t)=\sum_{i=1}^{N}\nu_i(t)(A_i(.)x(t)+B_i(t)u(t)) \tag{4}$$

where $\nu_i(t)$ is defined in (3) and $\sum_{i=1}^{N}\nu_i(t)=1$, $\forall t\ge 0$.

2.2. Preliminaries

Now, the following definitions, lemma, criterion and theorem are preliminarily presented for later development.

Definition 1. The autonomous switched nonlinear system (1), $(u(t)=0)$ is said to be stable if for any $\varepsilon>0$ there exists $\delta=\delta(\varepsilon)>0$ such that $\|x(t)\|\le\varepsilon$ for all $t\in[t_0\infty)$ whenever $\|x(t_0)\|\le\delta$.

Furthermore, the autonomous switched nonlinear system (1) is asymptotically stable if $x(t)\to 0$ as $t\to\infty$ for arbitrary switching signal $\sigma(t)$.