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Finite time stability of nonlinear impulsive systems and its applications in sampled-data systems

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ABSTRACT

In this paper, we establish finite time stability (FTS) criteria for the nonlinear impulsive systems. By using a new concept called average impulse interval (AII), less conservative conditions are obtained for the FTS problem on the impulsive systems. Then we consider the linear time-invariant sampled-data systems by modeling such systems as linear impulsive systems. It is proved that when the AII of a sequence of impulsive signals ζ is equal to τ_α, the upper bound of the impulsive intervals could be very large, while the lower bound of the impulsive intervals could be also small enough. The obtained results are less conservative than the ones in the literature obtained for variable sampling intervals.

1. Introduction

The concept of finite time control dates back to the Sixties, when the idea of finite time stability (FTS) has been first introduced in the control literature. In order to deepen our understanding of FTS, it is necessary to make a distinction between FTS and finite time boundedness (FTB). A system is said to be finite time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval, while FTS in the presence of exogenous inputs leads to the concept of FTB. In other words, a system is said to be finite time bounded if, given a bound on the initial condition and a characterization of the set of admissible inputs, the state variables remain below the prescribed limit for all inputs in the set (see [1]). Often asymptotic stability is enough for practical applications, but there are some cases where large values of the state are not acceptable, for instance in the presence of saturations. In these cases, we need to check that these unacceptable values are not attained by the state. FTS can then be used for these purposes. In the past decades, much work has been done for the FTS [2]. In [1,3,4], the FTS problem of *linear time-invariant* systems has been solved, and a feedback controller has been designed. When referring to the *linear time-varying* systems, many criteria have been given in [5–9]. Furthermore, L_2-gain problem based on FTS has been studied in [36,37]. In [38], finite-time stabilization problem of switch linear system with nonlinear saturating actuators is considered. State feedback controllers are designed to show the effect of the switching signals on finite-time stabilization of the system. On the other hand, as an effective robust control strategy, sliding mode control has been considered in [40]. Especially in [40], new condition for sliding mode control based on observer is built.

Although much work has been done on FTS, the works on the nonlinear systems are few [10]. As far as we know, no general principles have been summarized which can provide useful reference for scholars to discuss the FTS topics. In this paper, we study the FTS problem for the nonlinear systems with nonlinear impulses. A sufficient condition is to be given which guarantees the nonlinear system to be finite time stable. A less conservative condition is obtained when referring to the impulsive times. In many papers such as [9,11–16], when discussing impulses, the authors usually require that $V(t_k^+, x(t_k^+)) \le V(t_k, x(t_k))$ (here, the Lyapunov function is assumed to be left continuous on the intervals $(t_{k-1}, t_k]$). Meanwhile, there are also many references such as [17,18] where the impulses are assumed to be satisfied $V(t_k^+, x(t_k^+)) > V(t_k, x(t_k))$. However, when dealing with such a case, they always assume that, at *each* impulsive time t_k, the condition $t_{k+1} - t_k \ge \eta$ is satisfied for some constant η. Therefore, much work has to be done to find the lower bound η, see Refs. [13,17] for more details. Now, a natural problem arises: Could the lower bound η be sufficiently small? In this paper, we give a positive answer to this question with the help

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of average impulse interval (AII). It is concluded that as long as the AII τ_α satisfies some condition, the upper boundary and lower boundary of interval between t_k and t_{k+1} could be either large enough or quite small respectively.

As an application of the FTS result obtained for the nonlinear systems with nonlinear impulses, we consider the sampled-date control problem for linear systems, which has been well-developed in the past decades. There are three main approaches to be utilized for the sampled-data stabilization. The first one is based on the lifting technique [19] where the problem is transformed into an equivalent finite-dimensional discrete problem [20–22]. However, this method is generally ineffective when dealing with the system with uncertain matrices or uncertain sampling times. For example, in [23] when dealing with the time-varying sampling intervals, it is assumed that the sampling interval must have a known upper bound. Therefore, many researchers intend to find new techniques to investigate the sampled-data systems. Lately, the time-delay approach occurs. More specifically, the sampled-data system is modeled as a continuous-time system with a delayed control input, and stability and stabilization conditions then can be derived by using the Razumikhin or Lyapunov–Krasovskii theorems. Furthermore, the requirement on the upper bound of the sampled intervals could even be abrogated.

In [24], a new approach to robust sampled-data control is introduced: the system is modeled as a continuous-time one with the control input having a piecewise-continuous delay. An improved stability condition has been derived in [25] for the sampled-data feedback control systems with uncertain time-varying sampling intervals. The stability and stabilization problems of aperiodic sampling control system have been investigated in [31] via the robust linear matrix inequality methods. One of the main works in these papers is to find a bigger upper bound of the sampling intervals since when dealing with the sampled-data system, strict conditions have to be restricted on the sampling intervals. For example, in [26] it is assumed that the length of the sampling interval must be less than a given constant h; and in [27–29], it is required $t_{k+1}-t_k\in[h_1,h_2]$, where t_k and t_{k+1} are the sampling times and h_1, h_2 are known constants.

The third one usually employs the impulse approach where the sampled-data system is treated as an impulsive system, and then the Lyapunov stability theory is used to study the dynamical behavior of the system. A lot of work has been done with this method (see Refs. [30,31] for example). In [18], the impulsive model approach has been investigated for the systems with variable sampling intervals with known upper bound. The existed results have been improved based on the input delay approach via the time-independent Lyapunov functional. Moreover, the obtained conditions distinguish between the cases of constant and variable sampling intervals and provide less conservative results for the constant sampling. Similar with the time-delay approach, the main purpose of the impulse approach is also to find the upper bound of the sampling intervals.

The method used here is also based on the impulsive modeling of the sampled-data systems. We choose a Lyapunov function to discuss the nature of the sampled-data system. By using the conception of AII, we not only study the case that $V(t_k^+,x(t_k^+))>V(t_k,x(t_k))$ at the sampling time t_k, but also prove that as long as the AII τ_α is fixed, the interval between t_k and t_{k+1} could be either very large or small enough, which means a big progress is to be made on the problem of finding the upper bound of the sampling intervals. The main contributions of the paper are as follows: Firstly, based on average impulse interval, less conservative sufficient conditions for the finite time stability of nonlinear impulsive systems is obtained; secondly, less conservative conditions for the sampling intervals of sampled-data systems to guarantee the finite time stability of linear sampled-data controlled systems is given. That is, when the AII of a sequence of impulsive signals ζ is equal to τ_α, the upper bound of the sampled intervals could be very large, while the lower bound of the sampled intervals could be also small enough.

The paper is organized as follows: Section 2 formulates the problem and establishes some useful notations. The main results are given in Section 3. Numerical examples are provided in Section 4 to illustrate the obtained results. Section 5 concludes the paper.

2. Preliminaries

2.1. Notations

In this paper, $R^+=[0,+\infty)$. $\|\cdot\|$ denotes the Euclidean norm. x' (respectively, A') represents the transpose of vector x (respectively, matrix A). $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stand for, respectively, the maximum and the minimum of the eigenvalues of matrix A. $\mathbf{0}$ is a matrix with compatible dimensions where the elements are all equal to 0's.

2.2. Problem formulation and some definitions

Consider the following impulsive system:

$$\begin{cases}\dot{x}(t)=f(t,x(t)), & t\neq t_k\\ x(t_k^+)=g(t_k,x(t_k)),\end{cases}\tag{1}$$

where $k\in\{1,2,3,\ldots\}$ and $x\in R^n$ denotes the state. $f(\cdot,\cdot)$: $R^+\times R^n\mapsto R^n$ and $g(\cdot,\cdot)$: $R^+\times R^n\mapsto R^n$ are locally Lipschitzian functions with $f(t,0)=0$ and $g(t,0)=0$ for $t\in R^+$. The time sequence $\{t_k\}$ $(k\in N)$ has the effect that the states of system (1) are suddenly changed at the fixed points $\{t_k\}$ with

$$0<t_1<t_2<\cdots<t_k<\cdots$$

and $\lim_{k\to\infty}t_k=\infty$. Here we assume that the state variables of system (1) are left continuous for each t_k, i.e., $x(t_k)=x(t_k^-)$; in other words, $x(t)$ is continuous at interval $(t_k,t_{k+1}]$.

Firstly, we introduce some basic definitions which will be used in the following paragraphs.

Definition 1 (*Amato et al. [9]*). Given positive numbers T, N_1 and N_2 with $N_2>N_1$, system (1) is said to be finite-time stable with respect to (T,N_1,N_2) if $\|x(0)\|\le N_1\Rightarrow\|x(t)\|<N_2,\forall t\in[0,T]$.

In this paper, we will give a sufficient condition for the FTS of the impulsive system (1) by using the AII approach. For this purpose, let us recall the definition of AII.

Definition 2 (*Lu et al. [32]*). The average impulsive interval of the impulsive sequence $\zeta=\{t_1,t_2,\ldots\}$ is equal to τ_α if there exist positive integer N_0 and positive number τ_α, such that

$$\frac{T-t}{\tau_\alpha}-N_0\le N_\zeta(T,t)\le\frac{T-t}{\tau_\alpha}+N_0,\quad\forall T\ge t\ge 0$$

where $N_\zeta(T,t)$ denotes the number of impulsive times of the impulsive sequence ζ on the interval $(t,T]$.

In this paper, we are interested in the behaviors of system (1) within a finite time interval $[0,T]$. Hence it is assumed that for the given time $T>0$, there exists a scalar $N_\zeta(T,0)\in N$ such that

$$0<t_1<t_2<\cdots<t_{N_\zeta(T,0)}\le T.$$

Remark 1. The concept of AII has been firstly introduced in [32], which has been shown to be an effective tool to deal with the non-uniformly distributed impulses. Fig. 1 presents a specific example of a non-uniformly distributed impulsive sequence with the average impulsive interval $\tau_\alpha=0.5$, $N_0=10$ and $T=10$, $(10-0)/0.5-10=10\le N_\zeta(10,0)=23\le 30=(10-0)/0.5+10$. It can be seen from Fig. 1 that for such impulsive sequence ζ, the lower bound of the impulsive intervals can be very small; meanwhile the upper bound of the impulsive intervals can be quite large. Hence, the