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Robust evolutionary particle filter

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A B S T R A C T

The particle filter (PF) has been widely applied for non-linear filtering owing to its ability to carry multiple hypotheses relaxing the linearity and Gaussian assumptions. However, PF is inconsistent over time due to the loss of particle diversity caused mainly by the particle depletion in resampling step and incorrect a priori knowledge of process and measurement noise. To overcome these problems, in this paper, robust evolutionary particle filter is proposed. The proposed method can work in unknown statistical noise and does not require a prior knowledge about the system. In addition, to increase diversity, a resampling process is done based on the differential evolution (DE). The effectiveness of the proposed algorithm is demonstrated through Monte Carlo simulations. The simulation results demonstrate the effectiveness of the proposed method.

1. Introduction

The state estimation problem has many applications in many fields, including signal processing, biostatistics, economics, and engineering. The aim of this problem is to find the actual values of states of a dynamical system using a sequence of noisy measurements [1]. The particle filter is an effective estimator for the nonlinear filtering problem. It generalizes the traditional Kalman filters and can be applied to nonlinear and non-Gaussian state-space models. PF is a Monte Carlo approach for implementing recursive Bayesian estimation [2–4]. It exploits some random particles with associated weights to approximate the true posterior density function [4,5]. The particles are evolved over time via a combination of importance sampling and resampling step.

To improve the performance of PF, choosing the proposal distribution and the resampling scheme is important. For this purpose, many variants of the particle filter have been reported. For example, the auxiliary particle filter [6], the regularized particle filter [7,8], and the bootstrap particle filter [9,10]. In [11,12], the extended Kalman filter (EKF) Gaussian approximation is used as the proposal distribution for PF, and in [13,14], the EKF proposal is replaced by an unscented Kalman filter (UKF) proposal, and the unscented particle filter (UPF) is proposed. However, there are two main sources for the inconsistency of UPF: first, the performance of UPF depends on correct a priori knowledge of the process and measurement noise that are unknown in real-life applications. An incorrect a priori knowledge may seriously degrade the performance of UKF [15–17]. It can even lead to practical divergence and inconsistency [12–14]. A classical method for solving this problem is adaptive estimation of a priori knowledge. Several research works have been reported in this direction, which have attempted adaptive estimation of a priori knowledge. In [18], an adaptive estimation of noise covariance matrices for Kalman filter based on the correlation–innovations method is reported. Another method for solving this problem is H-infinity filtering [19,20]. Compared to the Kalman filter that requires an exact and accurate system model as well as a perfect knowledge of the noise statistics, the H-infinity filtering requires no a priori knowledge of the noise statistics [21–23]. In particular, unlike the Kalman filter that aims to give the minimum mean-square estimate, the H-infinity filtering minimizes the effect of the worst possible disturbances on the estimation errors and hence it is more robust against model uncertainty [24,25]. However, these results are limited to linear systems. The extended H-infinity filter (EHF) is proposed for non-linear systems in [19–22]. However, EHF also suffers from a number of drawbacks, namely the problem caused by derivation of the Jacobian matrices and the linear approximations of the nonlinear functions [13,14].To overcome the limitations of EHF, the unscented transformation (UT) technique has been combined with H-infinity filter because of its effectiveness in addressing non-linear state estimation problems. The unscented transformation (UT) is an elegant way to approximate the filtering distribution by a Gaussian density instead of approximating the non-linear functions as EKF. It has been shown that the UT-based estimates are accurate to the second order of the Taylor series expansion compared to EKF and hence perform better than that of EKF. However, one of the most costly operations in UPF is the calculation of the square root of the state variable covariance matrix each time. In addition, although

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H-infinity techniques have been studied in many fields such as EKF [22–26] and UKF [27], there are few studies on H-infinity filtering in PF [28]. Second, UPF uses the classical resampling technique to avoid the particle degeneracy. During resampling, some particles might end up with no children, whereas others might end up having a large number of children.

Both limitations can seriously reduce the accuracy of UPF and can decrease the particle diversity, which would lead to inconsistency [29–31]. In this paper, square root unscented H-infinity PF with the evolutionary resampling to overcome these problems is presented. The proposed algorithm uses the H-infinity square root unscented Kalman filter to generate the importance density. As a result, it can work in unknown statistical noise behavior and is more robust. In addition, besides the merit of reducing the computational complexity, it has other advantages such as consistently increasing numerical stability and better performance because all resulting covariance matrices are guaranteed to stay semi-positive definite. In the proposed algorithm, the differential evolution based resampling scheme called evolutionary resampling is used. In the evolutionary resampling scheme, the particles recombine by using an iterative process of mutation, crossover and selection and unlike the traditional resampling schemes; there is no duplication and elimination of particles.

The rest of the paper is organized as follows. In Section 2, the required background is reviewed and the consistency analysis of UPF is presented. The H-infinity square root Unscented Kalman filter is introduced in Section 3. The square root unscented H-infinity PF with evolutionary resampling is presented in Section 4. In Section 5, the experimental results are compared and analyzed with simulation results.

2. Background

2.1. Particle filter

Consider the following nonlinear discrete system:

$$x_t = f(x_{t-1}) + w_t$$
$$y_t = h(x_t) + v_t \tag{1}$$

where $x_t \in R^n, y_t \in R^m$ are the state vector and the measurement vector, respectively. $f(.), h(.)$ are known nonlinear function with appropriate dimensions and w_t, v_t are the process noise and measurement noise, respectively. The process noise w_t and the measurement noise v_t are assumed to be mutually uncorrelated zero mean white noise processes with covariance matrices Q and R, respectively. It should be pointed out that the statistics of noise processes might not be known by the user to design filtering algorithms. The objective of filtering is to estimate the posterior density of the state given the past measurements $p(x_t|y_{1:t})$ [2,3,10]. Generic Bayesian filtering is used to estimate the state of a nonlinear dynamic system sequentially in time. A recursive update of the posterior density as new measurement arrive is given by the recursive Bayesian filter defined by

$$p(x_t|y_{1:t-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|y_{1:t-1})dx_{t-1}$$
$$p(x_t|y_{1:t}) = \frac{p(y_t|x_t)p(x_t|y_{1:t-1})}{p(y_t|y_{1:t-1})} \tag{2}$$

where the conditional density $p(y_t|y_{1:t-1})$ is as follows:

$$p(y_t|y_{1:t-1}) = \int p(y_t|x_t)p(x_t|y_{1:t-1})dx_t. \tag{3}$$

The difficulty in implementing the recursive Bayesian filter is intractable integrations in estimating the posterior density, except for a linear Gaussian system. To solve these difficulties, probability densities in Bayesian filtering are represented by particle filtering. The generic PF uses the Monte Carlo simulation method to calculate the integrals. In PF, the posterior $p(x_t|y_{1:t})$ approximated as a weighted sum of singleton probability density functions is as follows [10]:

$$p(x_t|y_{1:t}) = \sum_{m=1}^{N} \omega_t^{[m]}\delta(x_t - x_t^{[m]}) \tag{4}$$

where $x_t^{[m]}, \omega_t^{[m]}$ are the position and weight of the mth particle respectively, $\delta(.)$ is Dirac's delta function, and N is the number of particles. The particle filter consists of three steps: sampling, importance weighting, and resampling.

In the sampling step, particles are sampled according to the proposal density function $q(x_t^{[m]}|x_{t-1}^{[m]}, y_t)$. The choice of the proposal density is one of the most critical issues in the particle filter, and two popular choices are the state transition, $p(x_t^{[m]}|x_{t-1}^{[m]})$ and $p(x_t^{[m]}|x_{t-1}^{[m]}, y_t)$.

In the importance weighting, the weights are updated as follows:

$$\omega_t^{[m]} = \omega_{t-1}^{[m]} \frac{p(y_t|x_t^{[m]})p(x_t^{[m]}|x_{t-1}^{[m]})}{q(x_t^{[m]}|x_{t-1}^{[m]}, y_t)} \tag{5}$$

In the resampling step, particles with low importance are deleted and replaced with high weights particles.

2.2. Consistency analysis

In general, a state estimator is called consistent if its state estimation errors satisfy following equations [32]:

$$E[x_t - \hat{x}_{t|t}] = E[e_t] = 0 \tag{6}$$

$$E([x_t - \hat{x}_{t|t}][x_t - \hat{x}_{t|t}]^T) = E[e_t e_t^T] = \hat{P}_{t|t} \tag{7}$$

where $\{\hat{x}_{t|t}, \hat{P}_{t|t}\}$ are the estimated mean and covariance of states on time step t, respectively. In the above equations, Eq. (5) is the unbiasedness requirement for estimates, while (6) is the covariance matching requirement. Ideally, in order to measure if a filter is consistent, one would compare its estimate with the probability density function (PDF) obtained from an ideal Bayesian filter. This is not practical when the true probability density function is not available. However, if the true state x_t is known, we can use the normalized estimation error squared (NEES) to carry out the consistency test [32]:

$$\varepsilon_t = (x_t - \hat{x}_{t|t})^T \hat{P}_{t|t}^{-1} (x_t - \hat{x}_{t|t}) \tag{9}$$

where x_t is the ground truth. Consistency is evaluated by performing multiple Monte Carlo runs and computing the average normalized estimation error squared (ANEES). Given N runs, ANEES is computed as follows:

$$\bar{\varepsilon}_t = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_{it} \tag{10}$$

Under the hypothesis that filter is consistent (called the hypothesis H_0), $\bar{\varepsilon}_t$ has a χ^2 distributed (chi-square distributed) and lies in the following interval:

$$\bar{\varepsilon}_t \in [r_1 \quad r_2] \tag{11}$$

where the acceptance interval $[r_1 \quad r_2]$ is determined such that:

$$P\{\bar{\varepsilon}_t \in [r_1 \quad r_2] | H_0\} = 1 - \alpha \tag{12}$$

in which α is the confidence level. The most common choice for confidence level is $\alpha = 0.05$. Therefore, the algorithm is consistent if $\bar{\varepsilon}_t$ belongs to $[r_1 \quad r_2]$ with probability 95% (i.e. $P\{\bar{\varepsilon}_t \in [r_1 \quad r_2]\} = 0.95$).

In UPF, parametric filter (unscented Kaman filter) is used for designing the proposal distribution. On the other hand, the