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Novel controller design for plants with relay nonlinearity to reduce amplitude of sustained oscillations: Illustration with a fractional controller

Ameya Anil Kesarkar, N. Selvaganesan *, H. Priyadarshan

Department of Avionics, Indian Institute of Space Science and Technology (IIST), Thiruvananthapuram-695547, India

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ABSTRACT

This paper proposes a novel constrained optimization problem to design a controller for plants containing relay nonlinearity to reduce the amplitude of sustained oscillations. The controller is additionally constrained to satisfy desirable loop specifications. The proposed formulation is validated by designing a fractional *PI* controller for a plant with relay.

1. Introduction

For systems containing separable nonlinearity in cascade, designed controllers usually produce undesirable sustained periodic oscillations in the plant output response due to the existence of stable limit cycles [1]. For the analysis of such limit cycles, Describing Function (DF) is a commonly used tool [2]. The studies in [3–5] show also the usefulness of DF for synthesis of controllers to satisfy the given limit cycle details. The work in [3] focuses on designing a robust limit cycle controller for a plant with relay nonlinearity by maintaining Nyquist plot of loop Transfer Function (TF) orthogonal to negative inverse of DF of relay nonlinearity. This is subsequently generalized for other nonlinearities in [4,5].

Motivated from the above works, we consider the problem of shaping the loop involving relay nonlinearity to reduce the amplitude of sustained oscillations. Additionally, we constrain the loop to meet certain performance specifications. For meeting such a stringent control requirement, we explore the potential of Fractional-Order Controllers (FOCs).

FOCs are the controllers whose dynamics are governed by fractional-order differential equations [6–8]. FOCs such as PI^{α}, $[PI]^{\alpha}$, PD^{β}, $[PD]^{\beta}$, $PI^{\alpha}D^{\beta}$ are superclass of their integer counterparts. Therefore, one expects them to perform better [9]. For instance, PI^{α} has the capacity to outperform integer-order *PI* controller. In the literature, the design of FOCs for linear plants has been widely studied [10–16]. In the limit cycle context, however, only a few works are seen for FOCs [17–19]. For the design problem in the current paper, we investigate the applicability of FOCs. The contributions of this paper are:

(i) An optimization problem is proposed to design a controller for plants containing relay nonlinearity in order to get:
  - Reduced sustained oscillation amplitudes.
  - Desirable loop performance.

(ii) Demonstration using fractional *PI* controller design.

2. Basics of relay nonlinearity and stable limit cycles

Let us consider a plant consisting of a relay nonlinearity in cascade with the TF $G(s)$. The closed loop control schematics with controller $C(s)$ is shown in Fig. 1.

Mathematically, the relay nonlinearity in Fig. 1 is given by

$$y_2(t) = \begin{cases} M & \text{if } y_1(t) \geq 0 \\ -M & \text{if } y_1(t) < 0 \end{cases} \tag{1}$$

We concentrate on a case where the Nyquist plot of the designed loop is assumed to intersect with negative inverse of

* Corresponding author. Tel.: +91 471 256 8456.
E-mail address: n_selvag@iist.ac.in (N. Selvaganesan).

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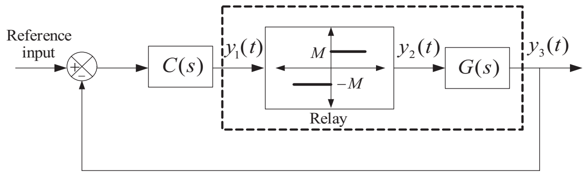

Fig. 1. Closed loop control schematics.

DF in the complex plane only once[1] as shown in Fig. 2. Fig. 2 contains superposition of:

(i) $-1/N(X)$ plot of relay nonlinearity. Where, $N(X)=4M/\pi X$ is its DF [1] and X is the peak amplitude of sinusoidal signal at the input of nonlinearity.
(ii) Nyquist plot of loop TF $L(s)=C(s)G(s)$.

The arrow in the Nyquist plot indicates increasing ω direction ($\omega \in [0,\infty)$). The arrow in $-1/N(X)$ plot shows increasing X direction. The limit cycle point ① is intersection point between Nyquist plot and $-1/N(X)$ curve.

For sustained oscillations to be produced at the output in the presence of nonlinearity, the nature of limit cycle point must be 'stable'. For the relay nonlinearity case, the occurrence and stability of limit cycles is ensured if the conditions explained below are met by the loop TF $L(s)$ (Refer Fig. 2).

1. Nyquist Condition for Limit Cycle Existence [1]:
$$\left[-\frac{1}{N(X)}\right]_{X=X_0}=[L(j\omega)]_{\omega=\omega_0} \tag{2}$$
where, ω_0 and X_0 are limit cycle frequency and amplitude at point ①.
2. Tsypkin's Condition [20] for Stability of Limit Cycle:
For stability of limit cycle, it is essential that for the given Nyquist curve seen in its arrow direction, the $-1/N(X)$ curve in its arrow direction crosses from right to left (refer [1,2] for details.). For relay nonlinearity case, this leads to the following Tsypkin's condition [20]:
$$\left[\frac{d}{d\omega}(\mathrm{Im}(L(j\omega))\right]_{\omega=\omega_0}>0 \tag{3}$$

3. A novel optimization problem for controller design

Many times, the presence of nonlinearity in the control loop shown in Fig. 1 produces undesirable sustained oscillations at the steady state due to the presence of stable limit cycles. It is usually desirable to design a controller which reduces the amplitude of sustained oscillations. Also, it will be advantageous if the controller can additionally meet certain loop specifications.

For the plant with relay nonlinearity, we intend the closed loop system to meet specified gain crossover frequency, phase margin and closed loop stability. In the following subsections, such design aspects are discussed to subsequently propose a constrained optimization problem.

3.1. Incorporating describing function to control transient behavior

In general, DF is used for computing amplitude and frequency of limit cycles, which sustain at the input of the nonlinearity. In the present subsection, we propose an additional application of DF for controlling the 'transient' behavior of a closed loop system in the presence of nonlinearity as follows:

- For the closed loop schematics shown in Fig. 1, one usually neglects the plant nonlinearity and considers only the TF $G(s)$ while designing the controller $C(s)$. This means that one assumes the nonlinearity to behave as a unity gain during the transient time. At a particular amplitude A of step reference, if the signal occurring at input of nonlinearity roughly takes shape of a sine-wave with peak amplitude P such that $N(P)=1$, then the transient performance of the designed closed loop control system with and without nonlinearity is 'same' for such A.

Remark 1. Limit cycles are the 'sustained' sine-waves that occur at the input of nonlinearity in the steady state. Whereas, we currently focus on a few cycles of sine-wave that occurs at the input of nonlinearity during the transient time. Therefore, P is different from limit cycle amplitude X_0.

- We extend the above concept for a general P ($N(P)\neq 1$) and design $C(s)$ for $N(P)G(s)$. For a particular step reference amplitude A, such designed control system meets the transient performance in the presence of plant nonlinearity. This is because, at such amplitude the input to the nonlinearity takes the form of sine-wave with amplitude P.
- It must be noted that the assumption of a sine-wave shaped input during the transient time is ideal, since input to the relay is not freely assigned. Therefore, it must be true only for a certain sub-class of loop TFs containing relay. Determining such a sub-class is a possible future direction to this work. Interestingly, we observe that a loop containing type-1 motion control plant and fractional *PI* controller satisfies such a sine-wave assumption as will be explained later in Section 4.2.

3.2. Desired loop specifications

Based on the discussion in the previous subsection, we consider the loop $C(s)N(P)G(s)$ to meet certain performance specifications. The specifications are as presented below (refer Fig. 3):

- Gain Crossover Frequency (ω_{gc}): $|C(j\omega_{gc})N(P)G(j\omega_{gc})|=1$.
- Phase Margin (ϕ_m): $\angle[C(j\omega_{gc})N(P)G(j\omega_{gc})]=-\pi+\phi_m$.

3.3. Proposed conditions for closed loop stability

To ensure the closed loop stability, necessary conditions need to be evaluated. One knows that the gain margin in decibels is $GM_{dB}=20\cdot\log_{10}(1/a)$ (refer Fig. 3). Therefore, for one requires $a<1$ for positive gain margin.

It is seen from Fig. 3 that $\angle[C(j\omega_{pc})N(P)G(j\omega_{pc})]=-\pi$. The relay nonlinearity does not introduce any phase shift, which results into its describing function $N(P)$ being a real quantity $(N(P)=4M/\pi P)$, i.e. $\angle N(P)=0$. It is also noticed from Fig. 2 that $\angle[C(j\omega_0)G(j\omega_0)]=-\pi$. Therefore, one can conclude that the limit cycle frequency ω_0 and phase crossover frequency ω_{pc} are equal for the case of relay nonlinearity. i.e.
$$\omega_0=\omega_{pc} \tag{4}$$
From Fig. 3, we have,
$$a=|C(j\omega_{pc})N(P)G(j\omega_{pc})|=N(P)|C(j\omega_{pc})G(j\omega_{pc})| \tag{5}$$

[1] Ref. [3] also focuses on such a specific case.