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Delay-dependent exponential passivity of uncertain cellular neural networks with discrete and distributed time-varying delays

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ABSTRACT

This paper is concerned with the delay-dependent exponential passivity analysis issue for uncertain cellular neural networks with discrete and distributed time-varying delays. By decomposing the delay interval into multiple equidistant subintervals and multiple nonuniform subintervals, a suitable augmented Lyapunov–Krasovskii functionals are constructed on these intervals. A set of novel sufficient conditions are obtained to guarantee the exponential passivity analysis issue for the considered system. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed results.

1. Introduction

As is well known, cellular neural networks (CNN) proposed by Chua and Yang in [1,2] have been extensively studied both in theory and applications. In order to take vagueness into consideration, parameter uncertainties are commonly encountered in modeling cellular neural networks due to the inaccuracies and changes in the environment of the model, and will break the stability of the systems. More specifically, the connection weights of the neurons are inherent, depending on certain resistance and capacitance values that inevitably bring in uncertainties during the parameter identification process. Deviations and perturbations in parameters are the main sources of uncertainty. So it is important to investigate the dynamical behaviors of cellular neural networks with uncertainty (see, e.g. [3–8] and the references therein).

On the other hand, the passivity theory was firstly related to the circuit theory that plays an important role in the analysis and design of linear and nonlinear systems, especially for high-order systems [9]. It should be pointed out that the essence of the passivity theory is that the passive properties of a system can keep the system internal stability. Very recently, the exponential passivity of neural networks with time-varying delays has been studied in [10–12], where sufficient conditions have been obtained for the considered neural networks to be exponential passivity. However, no authors take more attention to the distributed time-varying delays. And delay-dependent stability conditions, which contains information concerning time delay, are usually less conservative than delay-independent ones, especially for neural network with a small delay. The more general delay decomposition approach were studied in [16–19]. A piecewise delay method is firstly proposed. In this paper, in order to obtain some less conservative sufficient conditions, firstly, we decompose the delay interval $[-h_u, 0]$ into $[-h_u, -h_u/2]$ and $[-h_u/2, 0]$. Secondly, we decompose the delay interval $[-h_u/2, 0]$ into N equidistant subintervals. Furthermore, we choose different weighting matrices that is $[-h_u/2, 0] = \bigcup_{j=1}^{N}[-j\delta, -(j-1)\delta]$, where $\delta = h_u/2N$. Lastly, we decompose the delay interval $[-h_u, -h_u/2]$ into M nonuniform subintervals and some scalars satisfying $h_u/2 < h_0 < h_1 < \cdots < h_M = h_u$, on each subinterval, we choose different weighting matrices that is $[-h_u, -h_u/2] = \bigcup_{i=1}^{M} [-h_i, -h_{i-1}]$. Up to now, there are almost no results on the problems of delay-dependent exponential passivity of cellular neural networks with discrete and distributed time-varying delays, which motivates this work.

Motivated by the above discussions, we study the problem of delay-dependent exponential passivity for a class of cellular neural networks with discrete and distributed time-varying delays in this paper. The aim of this paper is to derive some new sufficient conditions for the systems. The method is based on a Lyapunov functional with novel delay decompose. We also provide two numerical examples to demonstrate the effectiveness of the proposed stability results.

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2. Model description and preliminaries

Consider the cellular neural networks with discrete and distributed time-varying delays described by the following state equations:

$$\begin{cases} \dot{x}(t) = -Ax(t) + B_0 g(x(t)) + B_1 g(x(t-h(t))) \\ \quad + B_2 \int_{t-\tau(t)}^{t} g(x(s))\, ds + u(t) \\ y(t) = g(x(t) + g(x(t-h(t))) + u(t) \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathcal{T}} \in R^n$ is the neuron state vector, $y(t) \in R^n$ is the output vector of neuron networks, n denotes number of neurons in a neural network, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^{\mathcal{T}} \in R^n$ denotes the activation function, $g(x(t-h(t))) = [g_1(x_1(t-h(t))), g_2(x_2(t-h(t))), \ldots, g_n(x_n(t-h(t)))]^{\mathcal{T}} \in R^n$, $A = diag(a_1, \ a_2, \ldots, a_n)$is a positive diagonal matrix, $B_0 = (b_{ij}^0)_{n\times n}, B_1 = (b_{ij}^1)_{n\times n}$, and $B_2 = (b_{ij}^2)_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $u(t) \in R^n$ is an external input vector to neurons.

The delays, $h(t)$ and $\tau(t)$ are time-varying satisfying

$$0 \le \tau(t) \le \tau_u, \quad 0 \le h(t) \le h_u, \quad \dot{h}(t) \le h_D.$$

The activation functions $g_j(x_j(t))$ $(j = 1, 2, \ldots, n)$ are assumed to be nondecreasing, bounded and globally Lipschiz that is,

$$0 \le \frac{g_j(\xi_1) - g_j(\xi_2)}{\xi_1 - \xi_2} \le l_j, \quad g_j(0) = 0, \tag{2}$$

and $\xi_1, \xi_2 \in R$, $\xi_1 \ne \xi_2 (j = 1, 2, \ldots, n)$. where $l_j > 0 (i = 1, 2, \ldots, n)$, we denote $L = diag[l_1, l_2, \ldots, l_n]$.

It is noted from [4] that $g_j(.)$ satisfies the following condition:

$$0 \le \frac{g_j(\xi_j)}{\xi_j} \le l_j, \quad \forall \xi_j \ne 0, \ j = 1, \ldots, n \tag{3}$$

In this paper, we deal the problem of exponential passivity for system (1). The proposed criterion is delay-dependent both on $\tau(t)$ and $h(t)$ and provides a convergence rate for guaranteeing the exponential passivity of system (1).

Before deriving our main results, we need the following definition and lemma.

Definition 2.1. The neural networks are said to be exponentially passive from input $u(t)$ to $y(t)$, if there exists an exponential Lyapunov function (or, called the exponential storage function) $V(x_t)$, and a constant $\rho > 0$ such that for all $u(t)$, all initial conditions $x(t_0)$, all $t \ge t_0$, the following inequality holds

$$\dot{V}(x_t) + \rho V(x_t) \le 2y^{\mathcal{T}}(t)u(t), \quad t \ge t_0 \tag{4}$$

where $\dot{V}(x_t)$ denotes the total derivative of $V(x_t)$ along the state trajectories $x(t)$ of system (1).

The parameter ρ provides an exponential convergence information about an upper bound of exponential Lyapunov function. If ρ increases, then tighter bound about Lyapunov function than the results in case of $\rho \ge 0$ can be provided.

Lemma 2.1 (Boyd et al. [14] Schur complement). *Given constant matrices Z_1, Z_2, Z_3, where $Z_1 = Z_1^{\mathcal{T}}$ and $Z_2 = Z_2^{\mathcal{T}} > 0$. Then $Z_1 + Z_3^{\mathcal{T}} Z_2^{-1} Z_3 < 0$ if and only if*

$$\begin{bmatrix} Z_1 & Z_3^{\mathcal{T}} \\ Z_3 & -Z_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -Z_2 & Z_3 \\ Z_3^{\mathcal{T}} & Z_1 \end{bmatrix} < 0.$$

Lemma 2.2 (Sun et al. [15]). *For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \to R^n$ such that the integrals concerned are well defined, then the following holds*

$$\left[\int_a^b x(s)\, ds\right]^{\mathcal{T}} M \left[\int_a^b x(s)\, ds\right] \le (b-a) \int_a^b x(s)^{\mathcal{T}} M x(s)\, ds.$$

Lemma 2.3. *For all real vectors a, b and all matrix $Q > 0$ with appropriate dimensions, it follows that:*

$$2a^{\mathcal{T}} b \le a^{\mathcal{T}} Q^{-1} a + b^{\mathcal{T}} Q b.$$

Lemma 2.4 (Zhang and Hang [13]). *For any constant matrix $X \in R^{n\times n}, X = X^{\mathcal{T}} > 0$,a scalar function $h : = h(t) > 0$, and a vector-valued function $\dot{x} : [-h, 0] \to R^n$ such that the following integrations are well defined:*

$$-h \int_{-h}^{0} \dot{x}(t+s)^{\mathcal{T}} X \dot{x}(t+s)\, ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^{\mathcal{T}} \begin{bmatrix} -X & X \\ * & -X \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}.$$

Lemma 2.5 (Zhu et al. [10]). *Given matrices $Q = Q^{\mathcal{T}}$, H, E and $R = R^{\mathcal{T}} > 0$ with appropriate dimensions, then $Q + HFE + E^{\mathcal{T}} F^{\mathcal{T}} H^{\mathcal{T}} < 0$ for all F satisfying $F^{\mathcal{T}} F \le R$, if and only if there exists an $\varepsilon > 0$ such that $Q + \varepsilon H H^{\mathcal{T}} + \varepsilon E^{\mathcal{T}} R E < 0$.*

3. Main results

In this section, we propose a new exponential passivity criterion for the cellular neural networks with discrete and distributed time-varying delays. Now, we have the following main results.

Theorem 3.1. *For given scalars $h_u > 0, \tau_u > 0, \rho > 0$ and $h_D < 1$, the delayed cellular neural networks (1) is exponential passive, if there exist positive diagonal matrices P_2, positive definite matrices $P_1, U, Q_j, M_j, R_j (j = 1, 2, \ldots, N)$, $\begin{bmatrix} X_i & Y_i \\ * & Z_i \end{bmatrix}$, W_i $(i = 1, 2, \ldots, M)$, N_1, N_2, and any positive diagonal matrices K_j $(j = 0, 1, 2, \ldots, N)$, K_t and S_i $(i = 0, 1, 2, \ldots, M)$ with appropriate dimensions such that the following LMIs hold*

$$\Phi_1 = \begin{pmatrix} \Delta + \Xi^1 & \Gamma & \Theta_1 \\ * & \Lambda & \Theta_2 \\ * & * & -R \end{pmatrix} < 0, \tag{5}$$

$$\Phi_2 = \begin{pmatrix} \Delta + \Xi^2 & \Gamma & \Theta_1 \\ * & \Lambda & \Theta_2 \\ * & * & -R \end{pmatrix} < 0, \tag{6}$$

where

$$\vartheta_{ij} = \begin{cases} -P_1 A - A P_1 + Q_1 - e^{-\rho\delta} R_1 + N_1 + \rho P_1, & i = j = 1, \\ e^{-\rho(i-1)\delta}(Q_i - Q_{i-1}) - e^{-\rho i \delta} R_i - e^{-\rho(i-1)\delta} R_{i-1}, & i = j = 2, 3, \ldots, N, \\ -e^{-\rho N\delta}(R_N + Q_N), & i = j = N+1, \\ -(1-h_D) e^{-\rho h_u} N_1, & i = j = N+2, \\ e^{-\rho h_0} X_1 - e^{-\rho h_1} W_1, & i = j = N+3, \\ e^{-\rho h_{i-N-3}}(X_{i-N-2} - X_{i-N-3}) \\ \quad - e^{-\rho h_{i-N-2}} W_{i-N-2} - e^{-\rho h_{i-N-3}} W_{i-N-3}, \\ & i = j = N+4, N+5, \ldots, N+M+2, \\ -e^{-\rho h_M}(X_M + W_M), & i = j = N+M+3, \\ e^{-\rho i \delta} R_i, & i,j = i+1, i, \ j \in \{1, 2, 3, \ldots, N-1\}, \\ e^{-\rho h_{i-N-2}} W_{i-N-2}, & i,j = i+1, i, \ j \in \{N+3, N+4, \ldots, N+M+2\}, \\ 0, & \text{otherwise}, \end{cases}$$