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Mode-dependent stochastic stability criteria of fuzzy Markovian jumping neural networks with mixed delays

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ABSTRACT

This paper investigates the stochastic stability of fuzzy Markovian jumping neural networks with mixed delays in mean square. The mixed delays include time-varying delay and continuously distributed delay. By using the Lyapunov functional method, Jensen integral inequality, the generalized Jensen integral inequality, linear convex combination technique and the free-weight matrix method, several novel sufficient conditions are derived to ensure the global asymptotic stability of the equilibrium point of the considered networks in mean square. The proposed results, which do not require the differentiability of the activation functions, can be easily checked via Matlab software. Finally, two numerical examples are given to demonstrate the effectiveness and less conservativeness of our theoretical results over existing literature.

1. Introduction

It is well known that many neural networks models have been extensively investigated and successfully applied to various areas such as signal processing, pattern recognition, associative memory and optimization problems [18,16,19,23,21,20,9]. In such applications, it is of prime importance to ensure that the designed neural networks are stable. In hardware implementation, time delays are likely to be present due to the finite switching speed of amplifiers and communication time. It has also been shown that the processing of moving images requires the introduction of delay in the signal transmitted through the networks. The time delays are usually variable with time, which will affect the stability of designed neural networks and may lead to some complex dynamic behaviors such as oscillation, bifurcation, or chaos. Therefore, the study of neural dynamics with consideration of time delays becomes extremely important to manufacture high quality neural networks.

As well known, in mathematical modeling of real world problems, we will encounter some other inconveniences, for instance, the complexity and the uncertainty or vagueness. Fuzzy theory is considered as a more suitable method for the sake of taking vagueness into consideration. Based on traditional neural networks, Yang et al. [14] introduced fuzzy cellular neural network in 1996, which combines fuzzy logic with the structure of traditional neural networks and maintains local connectedness among cells. Unlike previous neural network structures, fuzzy neural network has fuzzy logic between its template and input and/or output deciding the sum of product operation, which allows us to combine the low of fuzzy systems. Fuzzy neural network is a useful paradigm for image processing problems and Euclidean distance transformation. In addition, fuzzy neural network has inherent connection to mathematical morphology, which is a cornerstone in image processing and pattern recognition. In recent years, various interesting results on the stability and other behaviors of fuzzy neural network have been reported [2,5,10].

Markovian jump systems introduced in [6] are the hybrid systems with two components in the state. The first one refers to the mode which is described by a continuous-time finite-state Markovian process, and the second one refers to the state which is represented by a system of differential equations. And many researchers have made a lot of progress in Markovian jump control theory [2,7,3,22,24]. In [7], Li et al. established robust stability conditions of nonlinear delayed Hopfield neural networks with Markovian jumping parameters by the Takagi–Sugeno fuzzy model. However, to the best of our knowledge, up to today there is only one paper (see [5]) reported on the stochastic stability for fuzzy neural networks with Markovian jumping parameters. In [5], Han et al. proposed several LMI-based global exponential stability criteria in

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the mean square for a class of fuzzy cellular neural networks with time-varying delays and Markovian jumping parameters. But the stability of fuzzy Markovian jumping neural networks with continuously distributed delay has not been addressed in the previous literatures.

Motivated by the above discussion, in this paper our purpose is to present some new stochastic stability criteria for a class of fuzzy Markovian jumping neural networks with mixed delays in mean square. By using Jensen integral inequality, the generalized Jensen integral inequality [8], linear convex combination, linear matrix inequality (LMI) technique and the improved approximation method [13], several novel sufficient conditions are derived to ensure the global asymptotic stability of the equilibrium point of the considered networks in mean square. The proposed results, which do not require the differentiability of the activation functions, can be easily checked via Matlab LMI Toolbox. Finally, two numerical examples are given to demonstrate the effectiveness and less conservativeness of our theoretical results over existing literature.

Notation: Throughout this paper, let $\mathbb{Z}_+$ denote the set of positive integers, W^T and W^{-1} denote the transpose and the inverse of a square matrix W, respectively. $W>0(<0)$ denotes a positive (negative) definite symmetric matrix, I denotes the identity matrix with compatible dimension, $0_{m\times n}$ denotes the $m\times n$ zero matrix, the symbol "$*$" denotes a block that is readily inferred by symmetry. The shorthand $\mathrm{col}\{M_1, M_2, \ldots, M_k\}$ denotes a column matrix with the matrices $M_1, M_2, \ldots, M_k$. $\mathrm{diag}\{\cdot\}$ stands for a diagonal or block-diagonal matrix, $\mathbb{N}=\{1,2,\ldots,n\}$. For $\tau>0, \mathcal{C}([-\tau,0];\mathbb{R}^n)$ denotes the family of continuous functions $\phi$from $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|=\sup_{-\tau\le s\le 0}|\phi(s)|$. Moreover, let $(\Omega,\mathbb{F},\mathbb{P})$ be a complete probability space with a filtration $\{\mathbb{F}_t\}_{t\ge 0}$ satisfying the usual conditions and $\mathbb{E}\{\cdot\}$ representing the mathematical expectation. Denote by $\mathcal{C}^p_{\mathbb{F}_0}([-\tau,0];\mathbb{R}^n)$ the family of all bounded, $\mathbb{F}_0-$measurable, $\mathcal{C}([-\tau,0];\mathbb{R}^n)-$valued random variables $\xi=\{\xi(s): -\tau\le s\le 0\}$ such that $\sup_{-\tau\le s\le 0}\mathbb{E}|\xi(s)|^p<\infty$. $\|\cdot\|$ stands for the Euclidean norm; matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem description and preliminaries

Fuzzy recurrent neural network model with Markovian jump can be described by the following model:

$$\begin{cases} \dot{x}_i(t) &= -d_i(\eta(t))x_i(t)+\sum\limits_{j=1}^{n} a_{ij}(\eta(t))f_j(x_j(t)) \\ & \quad +\sum\limits_{j=1}^{n} b_{ij}(\eta(t))f_j(x_j(t-\tau(t,\eta(t)))) \\ & \quad +\sum\limits_{j=1}^{n} c_{ij}\varrho_j+\chi_i+\bigwedge\limits_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(x_j(s))\,ds \\ & \quad +\bigvee\limits_{j=1}^{n}\beta_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(x_j(s))\,ds+\bigwedge\limits_{j=1}^{n}\sigma_{ij}\varrho_j+\bigvee\limits_{j=1}^{n}\delta_{ij}\varrho_j, \\ x_i(s) &= \varphi_i(s), \quad s\in(-\infty,0], \quad i\in\mathbb{N}, \end{cases} \tag{1}$$

where $\alpha_{ij},\beta_{ij},\sigma_{ij}$ and δ_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $d_i(\eta(t))$ is a positive scalar representing the firing rate, $a_{ij}(\eta(t))$ and $b_{ij}(\eta(t))$ are elements of feedback template and c_{ij} are elements of feed-forward template. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively. $x_i(t)$, ϱ_j and χ_i denote state, input and bias of the ith neurons, respectively. $\{\eta(t), t\ge 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in finite set $\mathcal{N}=\{1,2,\ldots,N\}$ based on given probability space $(\Omega,\mathbb{F},\mathbb{P})$ and the initial model η_0. Let $\Pi=[\pi_{ij}]_{N\times N}$ denote the transition rate matrix with transition probability

$$\mathbb{P}(\eta(t+\delta)=j|\eta(t)=i)=\begin{cases}\pi_{ij}\delta+o(\delta), & i\ne j,\\ 1+\pi_{ii}\delta+o(\delta), & i=j,\end{cases}$$

where $\delta>0$, $\lim_{\delta\to 0^+}o(\delta)/\delta=0$ and π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij}\ge 0$ for $i\ne j$ with

$$\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}, \quad i,j\in\mathcal{N}.$$

$f_i(\cdot)$ is the activation function, $\tau(t,\eta(t))$ is the transmission delay. $k_j(s)\ge 0$ is the feedback kernel and satisfies

$$\int_0^{\infty}k_j(s)\,ds=1, \quad j\in\mathbb{N}. \tag{2}$$

Function $\varphi_i(s)(i\in\mathbb{N})$ is continuous on $(-\infty,0]$, the norm is defined by

$$\|\varphi\|_\infty=\max\left\{\sup_{-\infty<s\le 0}\|\varphi(s)\|, \sup_{-\infty<s\le 0}\|\dot{\varphi}(s)\|\right\}.$$

In this paper, we make the following assumptions.

(H1) The transmission delay $\tau(t,\eta(t))$ is time-varying and satisfies $0\le\tau(t,\eta(t))\le\tau(\eta(t))\le\tau$, $\dot{\tau}(t,\eta(t)))\le\tau'(\eta(t))<1$, where $\tau(\eta(t)),\tau,\tau'(\eta(t))$ are known constants.

(H2) The activation function $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^T\in\mathbb{R}^n$ is bounded and satisfies the following condition:

$$0\le\frac{f_j(s_1)-f_j(s_2)}{s_1-s_2}\le\lambda_j, \quad \forall\ s_1,s_2\in\mathbb{R},\ s_1\ne s_2,$$

where λ_j $(j=1,2,\ldots,n)$ are known real constants.

For simplicity, we denote $\Lambda=\mathrm{diag}\{\lambda_1,\lambda_2,\cdots,\lambda_n\}$.

For convenience, each possible value of $\eta(t)$ is denoted by $m(m\in\mathcal{N})$ in the sequel. Then we have

$$d_{im}=d_i(\eta(t)), \quad a_{ijm}=a_{ij}(\eta(t)), \quad b_{ijm}=b_{ij}(\eta(t)).$$

As well known, the Itô's formula plays important role in the stability analysis of Markovian systems and we cite some related results here [1]. Consider a general Markovian delay system

$$\dot{z}(t)=h(t,z(t),z(t-\kappa),\eta(t)), \tag{3}$$

on $t\ge t_0$ with initial value $z(t_0)=z_0\in\mathbb{R}^n$, where $\kappa>0$ is time delay, $h:\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathcal{N}\to\mathbb{R}^n$. Let $\mathcal{C}^{2,1}(\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathcal{N},\mathbb{R}^+)$ denote the family of all nonnegative functions $V(t,z,v,\eta(t))$ on $\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathcal{N}$ which are differentiable in t and continuously differentiable twice in z,v. Let $\mathfrak{L}$ be the weak infinitesimal generator of the random process $\{z(t),\eta(t)\}_{t\ge t_0}$ along the system (3) (see [11,15]), i.e.

$$\mathfrak{L}V(t,z_t,v_t,m):=\lim_{\delta\to 0^+}\frac{1}{\delta}[\mathbb{E}\{V(t+\delta,z_{t+\delta},v_{t+\delta},\eta(t+\delta))|z_t,v_t,\eta(t)=m\} \\ -V(t,z_t,v_t,\eta(t)=m)],$$

then, by the Dynkin's formula [24,17], one can get

$$\mathbb{E}V(t,z(t),v(t),m)=\mathbb{E}V(t_0,z(t_0),v(t_0),m)+\mathbb{E}\int_{t_0}^{t}\mathfrak{L}V(s,z(s),v(s),m)\,ds.$$

In addition, we use the following lemmas:

Lemma 1 (*See [12]*). *Let X,Y and P be real matrices of appropriate dimensions with $P>0$. Then for any positive scalar ε the following matrix inequality holds:*

$$X^TY+Y^TX\le\varepsilon^{-1}X^TP^{-1}X+\varepsilon Y^TPY.$$

Lemma 2 (*Jensen integral inequality, see [4]*). *For any constant matrix $M>0$, any scalars a and b with $a<b$, and a vector function $\chi(t):[a,b]\to\mathbb{R}$ such that the integrals concerned are well defined,*