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Trajectory-Shaping Guidance with final speed and load factor constraints

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ABSTRACT

This paper describes the design of a guidance law used for guiding a hypersonic gliding vehicle against a ground target from a near-vertical orientation with a specified final speed and a near-zero final load factor. The guidance law consists of two terms: one is Trajectory-Shaping Guidance (TSG) used for steering the vehicle to the target from the specified orientation; the other is Final-Speed-Control Scheme (FSCS) used for controlling the vehicle to perform lateral maneuver to adjust the final speed. Further, the generalized closed form solutions of TSG are obtained from a more general linearized engagement model, where the speed of the vehicle can be an arbitrary positive function of time. By analyzing these solutions, the stability domain of the guidance coefficients is obtained such that the final load factor is zero. This domain is not affected by the change rate of the speed. Thus, according to this analysis, the proposed guidance law can achieve a near zero final load factor by properly selecting the guidance coefficients in the stability domain.

1. Introduction

The terminal guidance problem of a hypersonic gliding vehicle [1] is studied in this paper. In this problem, in order to let the seeker have a good field of view, the guidance law needs to steer the vehicle to destination from a near-vertical orientation. Meanwhile, it is required that the guidance law can adjust the final speed, because if the final speed is too large, the constraints on heating rate and dynamic pressure would be violated, and if the final speed is too small, it is not conducive to breaking through the hostile defense system. Moreover, if the payload of the vehicle is an earth penetrator, it is desired that the vehicle hits the target with a small Angle Of Attack (AOA). If the final AOA is too large, a huge asymmetric force relative to the vehicle's axis of symmetry would act on the vehicle, which causes that the penetration path become curvilinear and the effective penetration depth is markedly reduced [2–4]. In fact, it is not easy to design a guidance law which can adjust the final AOA while keeping a small miss distance. However, we can design a guidance law whose acceleration command goes to zero finally, and thus achieve the goal of obtaining a small final AOA indirectly.

In 1964, Cherry [5] first put forward a Trajectory Shaping Guidance, named Explicit Guidance (E Guidance), by assuming that the acceleration command was a polynomial function of time. E Guidance was used to control the Apollo spacecraft to land on the moon with a desired final velocity vector. Now E Guidance is the typical representative of the Trajectory-Shaping Guidance laws. Bryson [6] derived E Guidance by solving the energy-optimizing problem. Lin [7] applied E Guidance to the flight vehicle whose control force was perpendicular to its velocity, and then optimized E Guidance coefficients with considering the aerodynamic and propulsion parameters. Zarchan [8] evaluated E Guidance in depth. In the later research, Ohlmeyer and Phillips [9] expanded the set of E Guidance coefficients using a new cost function that involves the integral of control energy divided by time-to-go to the nth power. Ben-Asher and Yaesh [10] augmented the E Guidance with an extra term to account for target maneuver. For powered vehicles, such as the Apollo spacecraft, E Guidance is able to control both the final speed and orientation, but for unpowered vehicles, E Guidance can only control the final orientation because the commanded acceleration along the velocity vector cannot be achieved. In [11–15], other types of Trajectory Shaping Guidance laws are presented. In [16], a novel guidance law is proposed for guiding missile against a maneuvering target while satisfying a circular no fly zone constraint. The key mechanism of this guidance law is to distort the real space such that the boundary of the no fly zone becomes a straight line.

At present, only reference-tracking guidance laws [17–20] can be used practically to handle the guidance problem studied in this paper. In such guidance laws, before the hypersonic vehicle is launched, a reference trajectory satisfying all constraints is planned using trajectory optimization methods [6,17,20–22], and stored in the guidance

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system. In the terminal guidance phase, the reference trajectory is followed by a tracking algorithm based on Linear-Quadratic Regulator (LQR) [17–19,23]. Reference-tracking guidance laws are effective and reliable if the state errors and external disturbances are small. To cope with the cases with large state errors and external disturbances, Oza [24] proposes an approach that updates the reference trajectory periodically in flight and then directly uses the results as the commands. However, in this approach, two new problems arise: 1) because the guidance problem is a strongly nonlinear problem with multiple constraints, the optimization calculation usually takes too much time, and thus it is possible that the computation time exceeds the time limit, and 2) sometimes, the planning process may be divergent. Both of these two problems may lead to mission failure.

In this paper, a new guidance law is proposed for steering a hypersonic gliding vehicle to destination from a near-vertical orientation with a desired final speed and a near-zero final load factor. The guidance law consists of Trajectory-Shaping Guidance (TSG) and Final-Speed-Control Scheme (FSCS). TSG can steer the vehicle to the target from the specified orientation, and FSCS can adjust the final speed by controlling the vehicle to perform lateral maneuver. Since the acceleration command of FSCS goes to zero as the vehicle gets close to the target, FSCS does not hinder TSG from steering the vehicle to the target. Further, in order that the proposed guidance law achieve a near zero final load factor, we study the TSG analytically. We get a linear time-varying system by linearization, and then obtain the closed form solutions of TSG. Different from the traditional closed form solutions [8], the speed of the vehicle in the new solutions can be an arbitrary positive function of time. So we name the new solutions the generalized closed form solutions. In the derivation of these solutions, we propose an innovative approach based on spectral decomposition [25] for solving a special type of linear time-varying system, in which the system matrix can be expressed as the product of a time-varying scalar function and a constant matrix. By analyzing the generalized solutions, the stability domain of TSG coefficients is obtained such that the acceleration command goes to zero finally. Because the proposed guidance law does not need a reference trajectory, it can deal well with the cases with large external disturbances and dispersions in initial states, and also allows the mission to be changed in flight. Therefore, the proposed guidance law is superior to the reference-tracking guidance laws.

In Section 7, to observe the influence of the response lag of the Flight Control System (FCS), wind and atmospheric density dispersion on the proposed guidance law, the simulations of 6 Degrees Of Freedom (DOF) motion are conducted where the vehicle is treated as a rigid body. In these simulations, the autopilots are used to control the attitude of the vehicle. The most commonly used autopilot in the missile flight control field is the so-called three loop autopilot [8] which can response quickly and track the command closely in the presence of disturbances and uncertainties. By the way, some other robust controllers and techniques [26–29] have the potential of being applied to the Flight Control System. In [26], a recursive technique was proposed for finding roots of equations in the presence of noisy measurements. In [27], the PI and PID parametric conditions guaranteeing the robust stability of the closed-loop systems were derived. In [28], a hybrid reference control with adaptive neuro-fuzzy inference system was proposed for improving transient response performance of the PID controller. In [29], a controller based on radial-basis-function neural network was proposed. This controller can easily adapt to different operation modes.

This paper is organized as follows: Section 2 shows the equations of motion; Section 3 gives the guidance law overview; Section 4 shows TSG, derives the generalized closed form solutions, and obtains the stability domain of the guidance coefficients; Section 5 shows FSCS in detail; Section 6 illustrates the model of CAV; Section 7 shows the performance of the proposed guidance law; Section 8 draws the conclusions, the references are listed finally.

2. Equations of motion

Fig. 1 illustrates the inertial frame of reference *o-xyH* and the state variables defined in this frame. *o-xyH* is fixed on the Earth. The vehicle is regarded as a particle and denoted by *M*. x is the downrange in meter. y is the crossrange in meter. H is the altitude in meter. V is the speed in m/s. γ is the flight-path angle in radian which is the angle between the velocity vector and the horizontal plane. γ is treated as positive when the altitude increases. ψ is the heading angle in radian which is the angle of the horizontal projection of the velocity vector measured counterclockwise from the x-axis. The equations of motion over the non-rotating flat Earth can be found in [30] as follows:

$$\frac{dx}{dt} = V\cos(\gamma)\cos(\psi) \tag{1}$$

$$\frac{dy}{dt} = V\cos(\gamma)\sin(\psi) \tag{2}$$

$$\frac{dH}{dt} = V\sin(\gamma) \tag{3}$$

$$\frac{dV}{dt} = -\frac{D}{m} - g\sin(\gamma) \tag{4}$$

$$\frac{d\gamma}{dt} = \frac{L\cos(\sigma)}{mV} - \frac{g\cos(\gamma)}{V} \tag{5}$$

$$\frac{d\psi}{dt} = -\frac{L\sin(\sigma)}{mV\cos(\gamma)} \tag{6}$$

where m is the mass in kg, σ is the bank angle in radian which is the angle between the lift vector and the vertical plane containing the current velocity vector, g is the gravitational acceleration, and L and D are lift and drag in Newton respectively. Eqs. (7–9) show the formulas of L, D, and g. The load factor is calculated by $n = L/(mg_0)$ where g_0 is the gravitational acceleration at sea level.

$$L = C_L q S_{ref} \tag{7}$$

$$D = C_D q S_{ref} \tag{8}$$

$$g = \frac{\mu}{(R_e + H)^2} \tag{9}$$

where C_L and C_D are the lift and drag coefficients respectively, $q = 0.5\rho V^2$ is the dynamic pressure in Pa, ρ is the atmospheric density in kg/m^3 and will be almost reduced by half if H increases by 4.6 km, S_{ref} is the reference area in m^2, μ is a constant of about 3.96272×10^{14} m$^3/s^2$, and R_e is the average radius of Earth with a value of 6356.766 km.

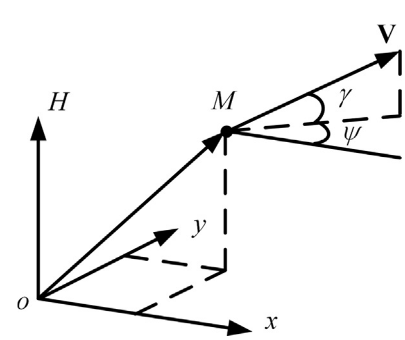

Fig. 1. Inertial reference frame *o-xyH* and the corresponding state variables.