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Improved robust stabilization method for linear systems with interval time-varying input delays by using Wirtinger inequality

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ABSTRACT

This paper investigates the robust stabilization problem for uncertain linear systems with interval time-varying delays. By constructing novel Lyapunov–Krasovskii functionals and developing delay-partitioning approaches, some delay-dependent stability criteria are derived based on an improved Wirtinger's inequality and the reciprocally convex method. The proposed methods have improved the stability conditions without increasing much computational complexity. A state feedback controller design approach is also presented based on the proposed criteria. Numerical examples are finally given to illustrate the effectiveness of the proposed method.

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1. Introduction

Recently, there has been rapidly growing interest in the stability of the system with time-varying delays, which has strong background in engineering field, such as a networked control system, see for example [1,2]. The Lyapunov–Krasovskii functional (LKF) approach is considered to be a powerful tool to derive effective criteria proving the stability of such a system. The derived delay-dependent criteria based on the LKF approach are usually expressed in forms of linear matrix inequalities (LMIs), whose conservatism is often judged by the upper bound of the time-varying delay. For a given lower bound, the larger the upper bound of the time-varying delay, the less conservatism the stability criteria.

Much effort has been invested in reducing the conservatism of the delay-dependent stability criteria of systems with time-varying delays, for example, the relaxation matrices method [1,3]; the convex analysis method [4]; the bounding estimation technology [5,6]; the delay-partitioning approach [1,7–9]; the augmented Lyapunov functional with some triple-integral terms method [10,11]; the reciprocally convex approach [12]; the novel integral equalities method [13]. A common feature of the aforementioned techniques is the use of slack variables and Jensen's inequality. Although Jensen's inequality has been recognized as a powerful tool to obtain efficient results, it generally induces some conservativeness difficult to overcome [14]. The free weighting matrices method has been shown an effective technique to improve the performance of delay-dependent criteria [1], but it increases the computational complexity with too much additional slack variables.

On the other hand, uncertain dynamics are usually unavoidable in practical systems, which may be attributed to modeling errors, measurement errors, parameter perturbations and a linearization approximation. Therefore, it is significantly important to take uncertainty into consideration for studying the stability of practical systems. In the past few decades, fruitful results have been obtained for the robust stability of uncertain systems with time-varying delays by using the LKF approach [13,15–21]. However, the reduced conservatism is often achieved at the price of much additional computational complexity in the aforementioned literature, which brings difficulty in the control synthesis problems.

In this paper, we revisit the stability of uncertain systems with time-varying delays by using an improved Wirtinger's inequality, which is an accurate integral inequality including Jensen's one as a special case [22]. Novel LKFs are introduced based on the Wirtinger inequality. Then refined delay-partitioning methods are proposed, which achieve less conservative results with fewer LMIs decision variables than some existing literature. Based on the proposed stability criteria, a state feedback controller design approach is developed for stabilizing the uncertain systems with time-varying

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input delays. Numerical examples are finally illustrated to demonstrate the effectiveness of the proposed methods.

Notation: $col\{\cdot\}$ denotes a column vector, * denotes a symmetric term in a matrix, $diag\{\cdot\}$ denotes the block diagonal matrix, the superscript T stands for matrix transposition, $P>0$ (≥ 0) means that P is positive definite (semi-positive).

2. Problem formulation and preliminaries

Consider the following system with time-varying input delay:

$$\begin{cases} \dot{x}(t)=(A+\Delta A(t))x(t)+(B+\Delta B(t))u(t-d(t)) \\ x(0)=x_0,\, u(t)=\phi(t),\, t\in[-h_2,0] \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ denotes the system state vector, $u(t)\in\mathbb{R}^m$ denotes the control input vector; A and B are known real constant matrices of appropriate dimensions; $\Delta A(t)$ and $\Delta B(t)$ are time-varying uncertain matrices satisfying

$$[\Delta A(t)\Delta B(t)]=MF(t)[E_aE_b], \tag{2}$$

where E_a and E_b are constant matrices of appropriate dimensions. $F(t)$ is an unknown time-varying matrix which is Lebesgue measurable in t and satisfies $F^T(t)F(t)\leq I$; $x_0\in\mathbb{R}^n$ and $\phi(t)\in\mathcal{C}[-\ h_2,0]$ are the initial conditions of the state and the input, respectively, where $\mathcal{C}[-h_2,0]$ denotes the set of continuous functions over $[-h_2,0]$; $d(t)$ denotes the input delay. As in [21], we consider the following two cases for $d(t)$.

Case 1: $d(t)$ is a continuous function satisfying

$$d(t)\in[h_1,h_2], \tag{3}$$

where h_1 and h_2 are constants and $0\leq h_1\leq h_2$.

Case 2: $d(t)$ is a differentiable function satisfying

$$d(t)\in[h_1,h_2],\quad \dot{d}(t)\leq\mu, \tag{4}$$

where h_1 and h_2 are as defined in Case 1 and μ is a constant.

Throughout this paper, the following assumption is needed.

Assumption 1. The full state variable $x(t)$ is available for measurement.

Under Assumption 1, we can design a state feedback control law as

$$u(t)=Kx(t), \tag{5}$$

where K is a constant matrix to be determined later.

According to the above arguments, system (1) can be written as

$$\begin{cases} \dot{x}(t)=Ax(t)+A_1x(t-d(t))+Mp(t) \\ p(t)=F(t)q(t) \\ q(t)=E_ax(t)+E_{a_1}x(t-d(t)) \\ x(t)=\phi(t),\, t\in[-h_2,0] \end{cases} \tag{6}$$

where $A_1=BK$, $E_{a_1}=E_bK$.

Remark 1. When the full states of system (1) are not completely measurable, the state feedback controller (5) is unavailable. Then we can consider other controllers such as an output feedback controller $u(t)=Ky(t)$, where $y(t)=Cx(t)$ and C is a constant matrix. In this case, $A_1=BKC$ and $E_{a_1}=E_bKC$. We will give some stability criteria for system (6) in the next section, which can be used for the controller design problem provided that the closed-loop system can be expressed as (6). For simplicity, we only consider the state feedback controller in this paper.

The following lemmas will be useful in the derivation of our main results.

Lemma 1 (*Seuret and Gouaisbaut [22]*). *For a given matrix $W=W^T>0$ and for any differentiable signal x in $[a,b]\to\mathbb{R}^n$, the following inequality holds:*

$$\int_a^b \dot{x}^T(s)W\dot{x}(s)\,ds\geq\frac{1}{b-a}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}^T\begin{bmatrix}W & 0\\ * & W\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix},$$

where

$$\omega_1=x(a)-x(b),$$

$$\omega_2=\frac{\pi}{2}x(a)+\frac{\pi}{2}x(b)-\frac{\pi}{b-a}\int_a^b x(s)\,ds.$$

Lemma 2 (Park et al. [12]). *Let $f_1,f_2,\ldots,f_N:\mathbb{R}^m\to\mathbb{R}$ have positive values in an open subset $\mathcal{D}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over $\mathcal{D}$ satisfies*

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i\alpha_i=1\}}\sum_i\frac{1}{\alpha_i}f_i(t)=\sum_i f_i(t)+\max_{g_{i,j}(t)}\sum_{i\neq j}g_{i,j}(t)$$

subject to

$$\left\{g_{i,j}:\mathbb{R}^m\to\mathbb{R},\, g_{i,j}\triangleq g_{i,j}(t),\,\begin{bmatrix}f_i(t) & g_{i,j}(t)\\ g_{i,j}(t) & f_j(t)\end{bmatrix}\geq 0\right\}$$

3. Stability criteria

In this section, we consider extending the delay-partitioning approach to the stability of system (6) based on the improved Wirtinger's inequality (Lemma 1).

3.1. A variable delay-partitioning approach to stability

Consider the delay interval $[h_1,h_2]$. In [1,7], the delay-central-point (DCP) method was used for dividing $[h_1,h_2]$. A generalized delay-partitioning approach was presented in [9], where the delay interval $[h_1,h_2]$ was divided into $N_v\geq 2$ equally spaced subintervals. However, with the addition of the number of delay-partitioning segments (N_v), both the number of decision variables and the number of LMIs are increasing dramatically. Moreover, the aforementioned methods are based on the idea of uniformly dividing the delay interval, which limits a further improvement of existing results.

In this section, we consider dividing the interval $[h_1,h_2]$ into two variable segments: $[h_1,\delta]$ and $[\delta,h_2]$, where δ is a positive scalar satisfying $h_1<\delta<h_2$. The proposed stability analysis is based on the following LKF:

$$V(x_t)=\sum_{i=1}^{4}V_i(x_t), \tag{7}$$

where

$$V_1(x_t)=\begin{bmatrix}x(t)\\ \int_{t-\delta}^t x(s)\,ds\\ \int_{t-h_2}^{t-\delta}x(s)\,ds\end{bmatrix}^T\begin{bmatrix}P & Q_1 & Q_2\\ * & R_1 & Q_3\\ * & * & R_2\end{bmatrix}\begin{bmatrix}x(t)\\ \int_{t-\delta}^t x(s)\,ds\\ \int_{t-h_2}^{t-\delta}x(s)\,ds\end{bmatrix}$$

$$V_2(x_t)=\int_{t-\delta}^t x^T(s)Z_1x(s)\,ds+\int_{t-h_2}^{t-\delta}x^T(s)Z_2x(s)\,ds$$

$$V_3(x_t)=\int_{t-h_1}^t x^T(s)S_1x(s)\,ds+\int_{t-d(t)}^{t-h_1}x^T(s)S_2x(s)\,ds$$

$$V_4(x_t)=h_1\int_{-h_1}^0\int_{t+s}^t\dot{x}^T(\theta)W_1\dot{x}(\theta)\,d\theta\,ds+(\delta-h_1)\int_{-\delta}^{-h_1}\int_{t+s}^t\dot{x}^T(\theta)W_2\dot{x}(\theta)\,d\theta\,ds+(h_2-\delta)\int_{-h_2}^{-\delta}\int_{t+s}^t\dot{x}^T(\theta)W_3\dot{x}(\theta)\,d\theta\,ds$$

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