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A design of a robust discrete-time controller

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ABSTRACT

In this paper, we proposed a robust discrete-time controller. This control system, which is derived from the idea of the normalized plant, does not include plant parameters. Thus, we obtain a control system independent of plant parameters and that has the same structure as a conventional optimal servo control system. Simulation and experimental results show that the proposed method is fairly robust to plant parameter variations and external disturbances.

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1. Introduction

The electric motor is a widely used device in cars, transportation equipment, robots, power stations and electrical household appliances. In recent years, electric motors have been recognized as one of the key components in hybrid and electric cars [1–4]. These devices have become large-scale and complicated, and require practical and stable operation [5–8]. When designing a control system for a mechanical system, the equation of motion is needed. However, it is difficult to obtain this equation, as it is affected by various uncertainties that exist in the actual system. When uncertainties are ignored in control system design, the mechanical system may become unstable. From this reason, a robust control system is required to allow for any uncertainties [9]. Various techniques for robust control systems have been proposed [5–8]. The optimal control method is one of the design methods used for robust control systems, and is an excellent method of control [10–12].

In recent years, this method has been applied in various fields [13,14], such as for precision-position control of hard disks and high-velocity revolution control of disk drives [15–17]. The optimal control design method is one of several design approaches that realize energy saving and input energy consumption as evaluation functions. However, this method may not produce the desired response when there are large changes to the plant parameters.

To avoid this problem, we previously designed a control system independent of plant parameters [18]. This method derived the linear control law from the switching function used in sliding mode control. This study demonstrates excellent robustness under plant parameter changes. However, it is not logically clarified that the control law is derived from the switching function. Furthermore, a logical proof of stability is also not clarified.

In this paper, we propose a control system based on a different point of view from the switching function and which does not include plant parameters. This is a new idea that we call the normalized plant. We analyze the obtained system and find that it is theoretically stable by using the z–s transform. The proposed method has the same structure as that of a conventional optimal servo controller and is easily mountable on actual systems such as a DC motor. In addition, the proposed method can eliminate steady-state errors caused by an input-side disturbance. We confirm the effectiveness of the proposed method using various simulations and experiments of position control with a DC motor.

This paper is organized as follows: In Section 2, we describe a plant in the discrete-time domain. In Section 3, we describe the structure of the normalized plant. In Section 4, we describe optimal control with a normalized plant. In Section 5, the effectiveness of the proposed method for five types of plants is confirmed by simulation results. In Section 6, the effectiveness of the proposed method for a DC motor is confirmed by experimental results. Finally, in Section 7, we present our conclusions.

2. Plant description

In this study, we assume that the controlled plant is a DC motor. In general, a DC motor has a constant numerator polynomial, and is expressed as first to third order plants. The general

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transfer function of a DC motor is described by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s^{n-1} + a_{n-1}s^{n-2} + \cdots + a_2 s + a_1}, \tag{1}$$

where $U(s)$ and $Y(s)$ are the Laplace transforms of $u(t)$ (the manipulated variable) and $y(t)$ (the plant output), respectively, and a_i $(i = 1, 2, \ldots, n-1)$ and b are plant parameters. Eq. (1) may also be described by

$$\dot{x}_i(t) = x_{i+1}(t), \quad i = 1, 2, \ldots, n-2, \tag{2a}$$

$$\dot{x}_{n-1}(t) = -\sum_{i=1}^{n-1} a_i x_i(t) + bu(t), \tag{2b}$$

$$y(t) = x_1(t). \tag{2c}$$

In addition, the derivative of the difference between a reference input and a plant output is described by

$$\dot{e}(t) = r(t) - y(t). \tag{3}$$

Eqs. (2) and (3) may be discretized by the forward difference method, shown in Eq. (4), and are then given as Eq. (5):

$$\dot{x}_i(t) = \frac{x_i(t+T_s) - x_i(t)}{T_s}, \tag{4}$$

$$e(k+1) = e(k) + T_s\{r(k) - y(k)\}, \tag{5a}$$

$$x_i(k+1) = x_i(k) + T_s x_{i+1}(k), \tag{5b}$$

$$x_{n-1}(k+1) = x_{n-1}(k) - \sum_{i=1}^{n-1} T_s a_i x_i(k) + T_s b u(k), \tag{5c}$$

where T_s is the sampling time. The expanded system including the deviation equation (5a) is described by

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}u(k) + \boldsymbol{E}r(k), \tag{6a}$$

$$y(k) = \boldsymbol{C}\boldsymbol{x}(k), \tag{6b}$$

where

$$\boldsymbol{x}(k) = \begin{bmatrix} e(k) \\ x_1(k) \\ \vdots \\ x_{n-2}(k) \\ x_{n-1}(k) \end{bmatrix} \in \mathcal{R}^{n\times 1},$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & -T_s & 0 & \cdots & 0 \\ 0 & 1 & T_s & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & T_s \\ 0 & -T_s a_1 & -T_s a_2 & \cdots & -T_s a_{n-1} \end{bmatrix} \in \mathcal{R}^{n\times n},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ T_s b \end{bmatrix} \in \mathcal{R}^{n\times 1}, \quad \boldsymbol{E} = \begin{bmatrix} T_s \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \in \mathcal{R}^{n\times 1}, \quad \boldsymbol{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^{\top} \in \mathcal{R}^{1\times n}.$$

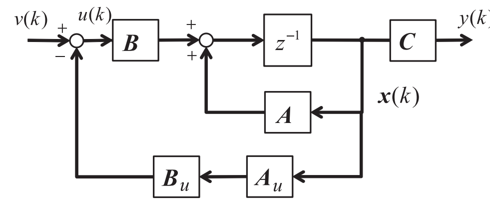

Fig. 1. Structure of the normalized plant.

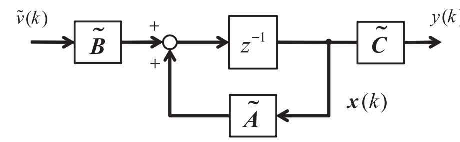

Fig. 2. Normalized plant.

3. Normalized plant

As shown in Fig. 1, the matrices in Eq. (7) are applied to the controlled plant as a state feedback:

$$\boldsymbol{A}_u = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & -T_s a_1 & -T_s a_2 & \cdots & -T_s a_{n-1} \end{bmatrix} \in \mathcal{R}^{n\times n}, \tag{7a}$$

$$\boldsymbol{B}_u = \begin{bmatrix} 0 & 0 & \cdots & 0 & \dfrac{1}{T_s b} \end{bmatrix} \in \mathcal{R}^{1\times n}. \tag{7b}$$

Therefore, the manipulated variable is described by Eq. (8) using the matrices $\boldsymbol{A}_u$ and $\boldsymbol{B}_u$. In addition, Eq. (6) is described by Eq. (9). This system is shown in Fig. 2, with $\tilde{v}(k)$ being a new manipulated variable:

$$u(k) = -\boldsymbol{B}_u\boldsymbol{A}_u\boldsymbol{x}(k) + v(k), \tag{8}$$

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) - \boldsymbol{B}\boldsymbol{B}_u\boldsymbol{A}_u\boldsymbol{x}(k) + \boldsymbol{B}v(k) \\ &= \tilde{\boldsymbol{A}}\boldsymbol{x}(k) + \tilde{\boldsymbol{B}}\tilde{v}(k), \end{aligned} \tag{9a}$$

$$y(k) = \boldsymbol{C}\boldsymbol{x}(k) = \tilde{\boldsymbol{C}}\boldsymbol{x}(k), \tag{9b}$$

where

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} 1 & -T_s & 0 & \cdots & 0 \\ 0 & 1 & T_s & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & T_s \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathcal{R}^{n\times n}, \tag{10a}$$

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathcal{R}^{n\times 1}, \quad \tilde{\boldsymbol{C}} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^{\top} \in \mathcal{R}^{1\times n}, \quad \tilde{v}(k) = T_s b v(k). \tag{10b}$$

The controlled system is described by Eq. (10) and does not include plant parameters. In this study, an optimal control method is adopted for this plant. We refer to this plant as the normalized plant.

4. Optimal control with the normalized plant

4.1. Derivation of optimal gain

In this section, we design an integral-type optimal controller. The expanded system including the deviation $e(k)$ is obtained from

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