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Further results on global state feedback stabilization of high-order nonlinear systems with time-varying delays

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ABSTRACT

This paper considers the problem of global stabilization by state feedback for a class of high-order nonlinear systems with time-varying delays. Comparing with the existing relevant literature, the systems under investigation allow more uncertainties, to which the existing control methods are inapplicable. By introducing sign function and necessarily modifying the method of adding a power integrator, a state feedback controller is successfully constructed to preserve the equilibrium at the origin and guarantee the global asymptotic stability of the resulting closed-loop system. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed approach.

1. Introduction

In this paper, we consider the following high-order time-delay nonlinear systems:

$$\dot{x}_i(t)=x_{i+1}^{p_i}(t)+f_i(\overline{x}_i(t),x_1(t-d_1(t)),\ldots,x_i(t-d_i(t))),$$

$$i=1,\ldots,n-1,$$

$$\dot{x}_n(t)=u^{p_n}(t)+f_n(\overline{x}_n(t),x_1(t-d_1(t)),\ldots,x_n(t-d_n(t))), \tag{1}$$

where $x(t)=(x_1(t),\ldots,x_n(t))^T\in R^n$ and $u(t)\in R$ are the system states and the control input, respectively. $\overline{x}_i(t)=(x_1(t),\ldots,x_i(t))^T$, $i=1,\ldots,n$, are state vectors. $d_i(t):R^+\to[0,d_i]$, $i=1,\ldots,n$, are the time-varying delays satisfying $\dot{d}_i(t)\le\eta_i<1$ for known constants d_i and η_i, the system initial condition is $x(\theta)=\varphi_0(\theta)$, $\theta\in[0,d]$ with $d=\max\{d_1,\ldots,d_n\}$. $p_i\in R_{odd}^{\ge 1}:=\{p/q|\ p$ and q are positive odd integers, and $p\ge q\}$, $i=1,\ldots,n$, are said to be the high orders of the system. f_i, $i=1,\ldots,n$, are unknown continuous functions.

Since one of the intrinsic features of system (1) is that the Jacobian linearization is neither controllable nor feedback linearizable, the existing design tools are hardly applicable to this kind of systems. Mainly thanks to the method of adding a power integrator, when $d_i(t)=0$, the global control design for system (1) has been well-studied and a number of interesting results have been achieved over the last decades, for example, one can see [1–12] and the references therein. When $d_i(t)\ne 0$, the global stabilization of system (1) is much more challenging because trade-off of time-delay effect and identification of time-delay restriction. Delightedly, to a certain extent, this problem has been solved in [13–17]. However, a common assumption of the above-mentioned results is that powers on the upper bound restrictions of nonlinearities are required to take values on some isolated points. Recently, the authors in [18,19] removed the restriction and achieved the feedback stabilization of system (1) under the following growth condition:

$$|f_i(\cdot)|\le a\sum_{j=1}^{i}\left(|x_j(t)|^{\nu_{hj}}+|x_j(t-d_j(t))|^{\nu_{hj}}\right) \tag{2}$$

where ν_{hj}'s take values continuously in interval $[1/p_j\cdots p_{i-1},+\infty)$. However, from both practical and theoretical points of view, it is still somewhat restrictive to require system (1) to satisfy such restriction. To illustrate the limitation, let us consider the following simple system:

$$\dot{x}_1=x_2$$
$$\dot{x}_2=u+x_1^{2/3}(t-d) \tag{3}$$

where $p_1=p_2=1$, $f_1=0$ and $f_2=x_1^{2/3}(t-d)$. It is easily verified that, even if $d=0$, the works [18,19] cannot lead to any continuous stabilizer for this system because f_2 dissatisfies the growth condition

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(2). Naturally, the following interesting questions are proposed: *Is it possible to further relax the nonlinear growth condition? Under the weaker condition, how can one design a state feedback stabilizer for the nonlinear system* (3) *and more general nonlinear system* (1).

In this paper, by introducing a combined sign function design and the method of adding a power integrator, we shall solve the above problems. The main contributions of this paper are twofolds: (i) By comparison with the existing results in [13–16,18,19], the nonlinear growth condition is largely relaxed and a much weaker sufficient condition is given. (ii) By successfully overcoming some essential difficulties such as the weaker assumption on the system growth, the appearance of the sign function and the construction of a continuously differentiable Lyapunov–Krasovskii functional, a new method to global state feedback stabilization of high-order time-delay nonlinear systems is given, which can lead to more general results never achieved before.

The remainder of this paper is organized as follows. Section 2 introduces some preliminaries and formulates the control problem. Section 3 presents the design scheme of robust stabilizing controller and the main results. Section 4 gives some extension of the results developed in Section 3. Section 5 provides two simulation examples to demonstrate the effectiveness of the theoretical results, and Section 6 gives some concluding remarks. The paper ends with an appendix.

2. Preliminaries and problem formulation

2.1. Preliminaries

Throughout this paper, the following notations are adopted. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. R^+_{odd}:={p/q| p and q are positive odd integers}, $R^{\geq 1}_{odd}$:={p/q| p and q are positive odd integers, and $p \geq q$}. For a given vector X, X^T denotes its transpose, and $|X|$ denotes its Euclidean norm. C^i denotes the set of all functions with continuous ith partial derivatives. $\mathcal{K}$ denotes the set of all functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; $\mathcal{K}_\infty$ denotes the set of all functions which are of class $\mathcal{K}$ and unbounded. A sign function sign(x) is defined as follows: sign(x) = 1, if $x>0$; sign(x) = 0, if x=0 and sign(x) = −1, if $x<0$. For any $a \in R^+$ and $x \in R$, the function $[x]^a$ is defined as $[x]^a = \text{sgn}(x)|x|^a$. Besides, the arguments of the functions (or the functionals) will be omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f.

We next provide three technique lemmas which will play an important role in the later control development.

Lemma 2.1 (Xie et al. [20]). *For $x \in R$, $y \in R$, $p \geq 1$ and $c>0$ are constants, the following inequalities hold: (i) $|x+y|^p \leq 2^{p-1}|x^p+y^p|$, (ii) $(|x|+|y|)^{1/p} \leq |x|^{1/p}+|y|^{1/p} \leq 2^{(p-1)/p}(|x|+|y|)^{1/p}$, (iii) $||x|-|y||^p \leq ||x|^p-|y|^p|$, (iv) $|x|^p+|y|^p \leq (|x|+|y|)^p$, (v) $|[x]^{1/p}-[y]^{1/p}| \leq 2^{1-1/p}|x-y|^{1/p}$, (vi) $|[x]^p-[y]^p| \leq c|x-y\|\,|x-y|^{p-1}+|y|^{p-1}|$.*

Lemma 2.2 (Yang and Lin [21]). *Let x, y be real variables, then for any positive real numbers a, m and n, one has*

$$ax^m y^n \leq b|x|^{m+n} + \frac{n}{m+n}\left(\frac{m+n}{m}\right)^{-m/n} a^{(m+n)/n} b^{-m/n} |y|^{m+n}$$

where $b>0$ is any real number.

Lemma 2.3 (Zhang and Xie [11]). *$f(x) = \text{sgn}(x)|x|^a$ is continuously differentiable, and $\dot{f}(x) = a|x|^{a-1}$, where $a \geq 1$, $\in R$. Moreover, if $x = x(t)$, $t \geq 0$, then $df(x(t))/dt = a|x|^{a-1}\dot{x}(t)$.*

2.2. Problem formulation

The objective of this paper is to design a state feedback controller of the form

$$u = u(x) \tag{4}$$

such that system (1) is globally asymptotically stable at the origin.

To this end, the following assumption regarding system (1) is imposed.

Assumption 2.1. For $i = 1, \ldots, n$, there are constants $a>0$ and $\tau > -1/\sum_{l=1}^n p_1 \cdots p_{l-1}$ such that

$$|f_i(\overline{x}_i(t), x_1(t-d_1(t)), \ldots, x_i(t-d_i(t)))|$$
$$\leq a \sum_{j=1}^{i} (|x_j(t)|^{(r_i+\tau)/r_j} + |x_j(t-d_j(t))|^{(r_i+\tau)/r_j}),$$

where $r_1 = 1$, $r_{i+1} = (r_i+\tau)/p_i > 0$, $i = 1, \ldots, n$ and $\sum_{l=1}^n p_1 \cdots p_{l-1} = 1$ for the case of l=1.

Remark 2.1. Assumption 2.1, which gives the nonlinear growth condition on the system drift terms, encompasses the assumptions in existing results [13–15,18,19]. Specifically, when $\tau = 0$, it reduces to Assumption 2.1 in [13]. When τ is some ratios of odd integers in $[0, +\infty)$, it becomes the condition used in [14,15]. Moreover, when $\tau \in [0, +\infty)$, it is equivalent to those in [18,19]. This means that the system studied in this paper is less restrictive and allows for a much broader class of systems.

3. Robust controller design

In this section, we proceed to explicitly construct a continuous state feedback controller by using the method of adding a power integrator.

Step 1: Let $\xi_1 = [x_1]^\sigma$, where $\sigma \geq \max_{1 \leq i \leq n}\{1, \tau+r_i\}$ is a positive constant and choose the Lyapunov–Krasovskii functional $V_1 = W_1 + (n/(1-\eta_1)) \int_{t-d_1(t)}^t \xi_1^2(s)\, ds + ((n-1)/(1-\eta_2)) \int_{t-d_2(t)}^t \xi_1^2(s)\, ds$, where $W_1 = \int_0^{x_1} [[s]^\sigma - 0]^{(2\sigma-\tau-1)/\sigma} ds$. With the help of Assumption 2.1 and Lemmas 2.1–2.3, we have

$$\begin{aligned}
\dot{V}_1 \leq\, & [\xi_1]^{(2\sigma-\tau-1)/\sigma} x_2^{p_1} + a[\xi_1]^{(2\sigma-\tau-1)/\sigma} \\
& \left(|x_1|^{1+\tau} + |x_1(t-d(t))|^{1+\tau}\right) + \frac{n}{1-\eta_1} x_1^{2\sigma} \\
& + \frac{n-1}{1-\eta_2}\xi_1^2 - \frac{n(1-\dot{d}_1(t))}{1-\eta_1}\xi_1^2(t-d_1(t)) \\
& - \frac{(n-1)(1-\dot{d}_2(t))}{1-\eta_2}\xi_1^2(t-d_2(t)) \\
\leq\, & [\xi_1]^{(2\sigma-\tau-1)/\sigma} x_2^{p_1} + \xi_1^2\left(a + l_1 + \frac{n}{1-\eta_1} + \frac{n-1}{1-\eta_2}\right) \\
& - (n-1)\left(\xi_1^2(t-d_1(t)) + \xi_1^2(t-d_2(t))\right),
\end{aligned} \tag{5}$$

where $l_1 = (2\sigma-\tau-1)/2\sigma \times ((1+\tau)/2\sigma)^{(1+\tau)/(2\sigma-\tau-1)} \times a^{2\sigma/(2\sigma-\tau-1)}$ is a positive constant.

Obviously, the first virtual controller x_2^* defined by

$$x_2^* = -\beta_1^{r_2/\sigma}[\xi_1]^{r_2/\sigma}, \tag{6}$$

with $\beta_1 \geq (n + a_1 + l_1 + n/(1-\eta_1) + (n-1)/(1-\eta_2))^{\sigma/r_2 p_1}$ being a constant, results in

$$\begin{aligned}
\dot{V}_1 \leq\, & -n\xi_1^2 - (n-1)(\xi_1^2(t-d_1(t)) + \xi_1^2(t-d_2(t))) \\
& + [\xi_1]^{(2\sigma-\tau-1)/\sigma}(x_2^{p_1} - x_2^{*p_1}).
\end{aligned} \tag{7}$$

Remark 3.1. It is necessary to mention that in the first design step, the constant l_1 has been provided with explicit expression in order to deduce the completely explicit virtual controller. However, in the later design steps, sometimes for the sake of briefness, we will not explicitly write out the constants which are easily defined.

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