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# Further improvement on delay-dependent robust stability criteria for neutral-type recurrent neural networks with time-varying delays

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ABSTRACT

This paper is concerned with the problem of improved delay-dependent robust stability criteria for neutral-type recurrent neural networks (NRNNs) with time-varying delays. Combining the Lyapunov–Krasovskii functional with linear matrix inequality (LMI) techniques and integral inequality approach (IIA), delay-dependent robust stability conditions for RNNs with time-varying delay, expressed in terms of quadratic forms of state and LMI, are derived. The proposed methods contain the least number of computed variables while maintaining the effectiveness of the robust stability conditions. Both theoretical and numerical comparisons have been provided to show the effectiveness and efficiency of the present method. Numerical examples are included to show that the proposed method is effective and can provide less conservative results.



## 1. Introduction

Over the recent decades there have been extensive investigates of neural networks (NNs) including Hopfield neural networks (HNNs), cellular neural networks (CNNs), recurrent neural networks (RNNs) and Cohen–Grossberg neural networks (CGNNs) [1,3–6,8,9,11–54]. Recently, considerable attention has been devoted to the study of artificial neural networks due to the fact that artificial neural networks can be applied in signal processing, static image treatment, and also can be applied to solve some image processing, pattern recognition and optimization problems [7,29,36,37]. Some of these applications require the uniqueness and asymptotic stability of the equilibrium point of a designed neural network. However, it is well known that, in the hardware implementation of recurrent neural networks, time delays inevitably occur in the signal communication among the neurons to lead to instability of the networks. Thus, the stability of recurrent neural networks with time delays has received much more attention both in theory and in practice [4,6,11–14,16,17,19,20,22,24,25,27,28,30,33–35,42,46–54].

In the past decade, most of research focus on the stability and robust stability of recurrent neural networks with retarded-type delay. There are a few reports on neutral-type neural networks (NTNNs) with time delay, i.e., recurrent neural networks with both retarded-type delay and neutral-type delays [5,6,12,19,21–23,27,31,35,41,43,50,51]. However, in Refs. [8,27], the time-delay is assumed to be constant, while it actually varies with respect to time in a physical system. Furthermore, the existing stability criteria for NTNNs with time delay rarely consider impact of neutral-type delay. In particular, NTNNs with fast-varying neutral-type delay (i.e., the derivative of delay is more than 1) can never be considered. These facts motivate the present studies.

In recent year, various approaches have been proposed to obtain stability criteria for time-delay neural networks. In Refs. [20,38], some sufficient conditions for global stability of neural networks have been provided, yet only constant delays are allowed in their results. But in practice, time-delay is usually time-varying, which can even largely change the dynamics of system in some cases. Therefore, the stability of neural networks with time varying delays has become more interesting than that of networks with constant time delays. Although stability criteria for neural networks with time-varying delay were derived in Refs. [3,15,21,22,32,33,45], the slow-varying constraints $\dot{h}(t)<1$ on time-varying delay was imposed. Such a restriction is very conservative and has physical limitations. Recently, He et al. [10,11] and Wu et al. [39,41] proposed a new method for dealing with time-delay systems, which employs free weighting matrices to express the relationships between the terms in the Leibniz–Newton formula, and all the negative terms in the derivative of the Lyapunov functional are retained. This approach avoids the restriction on the derivative of a time-varying delay. To the best of our knowledge, the problem of delay-dependent robust stability criteria for neutral-type recurrent neural networks (NRNNs) with time-varying delays has not been fully studied in the

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literature and still remains open. Motivated by the above-mentioned analysis, in this paper, by using an integral inequality approach (IIA) and linear matrix inequality (LMI) techniques, new delay-dependent criteria for the neutral-type recurrent neural network with time-varying delays and parameter uncertainties to be admissible are established.

Based on the above discussion, we discuss the neutral-type neural networks with time varying delays. The main purpose of this paper is to study the robust stability of the neural networks of the uncertain neutral type with time varying delays in terms of linear matrix inequalities (LMIs). The parameter uncertainties are assumed to be bounded in given compact sets and appear in all the matrices of the state-space model. The activation functions are supposed to be bounded and globally Lipschitz continuous, which are more general than the usual bounded monotonically increasing ones such as the activation functions of the sigmoid type. Attention is focused on the derivation of a sufficient condition which guarantees the existence, uniqueness and global asymptotic stability of the equilibrium point of the uncertain neutral neural network for all admissible uncertainties. The main advantage of the LMI based approaches is that the LMI stability conditions can be solved numerically using MATLAB LMI toolbox, which implements the state of art interior-point algorithms. We also provide numerical examples to demonstrate the effectiveness of the proposed stability results.

**Notations**: Throughout this paper, the superscripts '$-1$' and '$T$' stand for the inverse and transpose of a matrix, respectively; $R^{n\times n}$ denotes an $n$-dimensional Euclidean space; $R^{m\times n}$ is the set of all $m\times n$ real matrices; $P>0$ means that matrix $P$ is symmetric positive definite; for real symmetric matrices $X$ and $Y$, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite); $I$ is an appropriately dimensional identity matrix; $X_{ij}$ denotes the element in row $i$ and column $j$ of matrix $X$; The notation $*$ always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. Problem formulation

Consider the following neutral-type recurrent neural network with time-varying delays and parameter uncertainties:

$$\begin{aligned}&\dot u(t)-(D+\Delta D(t))\dot u(t-h)\\&\quad=-(C+\Delta C(t))u(t)+(A+\Delta A(t))f(u(t))\\&\qquad+(B+\Delta B(t))f(u(t-h(t)))+J,\end{aligned}\tag{1}$$

where $u(t)=[u_1(t),\ \dots,\ u_n(t)]^T\in R^n$ is the state vector with the $n$ neurons; $f(u(t))=[f_1(u_1(t)),\ \dots,\ f_n(u_n(t))]^T\in R^n$ is called an activation function indicating how the $j$th neuron responses to its input; $C=diag(c_1,\ \dots,\ c_n)$ is a diagonal matrix with each $c_i>0$ controlling the rate with which the $i$th unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, and $D=(d_{ij})_{n\times n}$ are interconnection matrices representing weight coefficient of the neurons; $J=[J_1,\ \dots,\ J_n]^T\in R^n$ is the external bias vector, $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$, and $\Delta D(t)$ are unknown matrices that represent the time-varying parameter uncertainties and $h(t)$ is the time delay of the system satisfies

$$0\leq h(t)\leq h,\quad \dot h(t)\leq h_d,\tag{2}$$

where $h$ and $h_d$ are some positive constants.

In this paper, the neuron activation functions are assumed to be bounded and satisfy the following assumption:

**Assumption 1.** [40]. It is assumed that each of the activation functions $f_j(j=1,\ 2,\ \dots,\ n)$ possess the following condition:

$$0\leq\frac{f_i(\varsigma_1)-f(\varsigma_2)}{\varsigma_1-\varsigma_2}\leq k_i,\quad \varsigma_1\neq\varsigma_2\in R,\quad i=1,\ 2,\ \dots,\ n,\tag{3}$$

where $k_i(i=1,\ 2,\ \dots,\ n)$ are known constant scalars.

We note that the existence of an equilibrium point of system (1) is guaranteed by the fixed point theorem [29]. Now letting $u^*=[u_1^*,\ \dots,\ u_n^*]^T$ be an equilibrium of (1), that is $\dot u^*(t)=0$, $\dot u^*(t-h)=0$, implies from (1) that

$$0=-(C+\Delta C(t))u^*(t)+(A+\Delta A(t))f(u^*(t))+(B+\Delta B(t))f(u^*(t-h(t)))+J,\tag{4}$$

Introducing the state deviation from equilibrium

$$x(t)=u(t)-u^*\tag{5}$$

where $x(\cdot)=[x_1(\cdot),\ \dots,\ x_n(\cdot)]^T$, with $g(x(\cdot))=[g_1(x_1(\cdot)),\ \dots,\ g_n(x_n(\cdot))]^T$, and

$$g_i(x_i(\cdot))=f_i(x_i(\cdot)+u_i^*)-f_i(u_i^*),g_i(0)=0,\quad i=1,\ 2,\ \dots,\ n.\tag{6}$$

Now subtracting (4) from (1) with some algebraic manipulations using (5) and (6), it is easy to see that the dynamics of the state deviation is governed by

$$\begin{aligned}\dot x(t)-(D+\Delta D(t))\dot x(t-h)=&-(C+\Delta C(t))x(t)+(A+\Delta A(t))g(x(t))\\&+(B+\Delta B(t))g(x(t-h(t))),\end{aligned}\tag{7}$$

It is obvious that the function $g_j(\cdot)(j=1,\ 2,\ \dots,\ n)$ satisfies the following condition:

$$0\leq\frac{g_i(x_i)}{x_i}\leq k_i,\quad g_i(0)=0,\quad \forall x_i\neq0,\quad i=1,\ 2,\ \dots,\ n,\tag{8}$$

which is equivalent to

$$g_i(x_i)(g_i(x_i)-k_ix_i)\leq0,\quad g_i(0)=0,\quad \forall x_i\neq0,\quad i=1,\ 2,\ \dots,\ n.\tag{9}$$

The matrices $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$, and $\Delta D(t)$ are the uncertainties of the system and have the form

$$\begin{bmatrix}\Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t)\end{bmatrix}=MF(t)\begin{bmatrix}N_a & N_b & N_c & N_d\end{bmatrix},\tag{10}$$

where $M$, $N_a$, $N_b$, $N_c$, and $N_d$ are known constant real matrices with appropriate dimensions and $F(t)$ is an unknown matrix function with Lebesgue-measurable elements bounded by

$$F^T(t)F(t)\leq I,\quad \forall t,\tag{11}$$

where $I$ is an appropriately dimensioned identity matrix.

The following lemmas are useful in deriving the criteria. First, we introduce the following integral inequality approach (IIA), which be used in the proof of ours.

**Lemma 1.** *[26,27]. For any positive semi-definite matrices*

$$X=\begin{bmatrix}X_{11} & X_{12} & X_{13}\\ * & X_{22} & X_{23}\\ * & * & X_{33}\end{bmatrix}\geq0\tag{12a}$$

*the following integral inequality holds:*

$$\begin{aligned}&-\int_{t-h(t)}^{t}\dot x^T(s)X_{33}\dot x(s)ds\\&\quad\leq\int_{t-h(t)}^{t}\begin{bmatrix}x^T(t) & x^T(t-h(t)) & \dot x^T(s)\end{bmatrix}\\&\qquad\times\begin{bmatrix}X_{11} & X_{12} & X_{13}\\ X_{12}^T & X_{22} & X_{23}\\ X_{13}^T & X_{23}^T & 0\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h(t))\\ \dot x(s)\end{bmatrix}ds.\end{aligned}\tag{12b}$$

Secondary, the following Schur complement result, which is essential in the proofs of Theorem 1, is introduced.