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Constrained off-line synthesis approach of model predictive control for networked control systems with network-induced delays

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ABSTRACT

This paper investigates the off-line synthesis approach of model predictive control (MPC) for a class of networked control systems (NCSs) with network-induced delays. A new augmented model which can be readily applied to time-varying control law, is proposed to describe the NCS where bounded deterministic network-induced delays may occur in both sensor to controller (S–A) and controller to actuator (C–A) links. Based on this augmented model, a sufficient condition of the closed-loop stability is derived by applying the Lyapunov method. The off-line synthesis approach of model predictive control is addressed using the stability results of the system, which explicitly considers the satisfaction of input and state constraints. Numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs) are control systems in which the control loop is closed over a real-time network [1–4]. The insertion of networks brings not only the advantages, but also some challenging problems, and conventional control theories for point-to-point control systems must be reevaluated before applying them to the NCSs. In the past few years, various methodologies have been proposed for modeling, stability analysis, and controller design for NCSs [5–11,33,26,27,15,14].

Network-induced delay is known as the major source of degrading the performance and even causing instability of NCSs. In order to overcome the adverse effect of network-induced delay in NCSs, the uncertain system approach [12,13], the impulsive system approach [16], the stochastic system approach [17–21,24,25], and time delay system approach [22] have been addressed in the literature. For data transmission existing in both sensor to controller (S–C) and controller to actuator (C–A) links, it is more difficult to study NCS with time-varying control law than that with time-invariant controller. One of the difficulties lies in the modeling of such NCS. For the time-invariant controller, the control law is static and one can establish the model of such NCS by combining the S–C and C–A delays, see [22,23]. However, for the time-varying control law, such as MPC, the controller works as an intelligent unit and calculates the control strategy based on the estimations of delayed steps of the current received data and the time of the control command arriving at the actuator, and hence, the approach of combining the delays in both links is not available. In the present paper, by respectively considering the bounded deterministic network-induced delays in S–C and C–A links, a novel augmented closed-loop model is provided which can be readily applied to MPC and other time-varying control laws.

MPC is one of the most popular control methods in industrial process control field, and the defining feature of MPC is handling the physical constraints in a systematic manner. A synthesis approach of MPC is the one which guarantees the closed-loop stability, i.e., the closed-loop system is stable whenever the optimization problem is feasible [36]. The synthesis approaches of MPC (see [28,29]) can significantly improve the control performance, especially when system uncertainties exist. The network-induced delays in NCS naturally introduce the uncertainties and hence, it is interesting and necessary to extend these approaches to the networked environment. Unfortunately, although there are many nice papers consider the stabilization of NCS with MPC (see, [30–32,34,37–39]), limited works have been found on the synthesis approaches of MPC. This situation motivates our present investigation.

This paper aims to give an off-line synthesis approach of MPC for NCS with network-induced delays in both S–C and C–A links. The most pertinent works to this paper are our previous works

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[35,36]. Ding [35] has provided the on-line synthesis approach of MPC for NCS with packet loss by extending the results in [28]. Tang and Ding [36] have investigated the on-line synthesis approaches of MPC for NCS where the packet loss and data quantization are coexisting, by generalizing the results in [35] and combing the sector bound approach in [33]. However, both works in [36,35] do not consider the network-induced delay which is considered as the major source of causing instability of NCSs. The reasons of preventing us to tackle the adverse effect of network-induced delays in the previous two papers lie in the following two aspects. First, since the property of network-induced delay process is completely distinct from that of packet loss, how to establish the model of such NCS needs to be further investigated. Actually, according to our study, the model of NCS with packet loss is only a special case of NCS with network-induced delays (case $\hat{x}=x$, where $\hat{x}$ is the received sensed state by the controller). The NCS model established in this paper incorporates the adverse effects of packet loss and network-induced delays simultaneously. Second, the different properties between the packet loss and network-induced delays make the main difficulty to obtain the synthesis approach of MPC. For example, in [35], the real-time condition on the augmented state of MPC, which is one of the crucial conditions in guaranteeing the recursive feasibility of synthesis approach of MPC, can be easily obtained by introducing "Assumption 9" that enables the controller to know whether or not the previous sent values have been applied. However, due to the properties of network-induced delays, the controller is difficult to know when the control law has been implemented and the initial condition cannot be easily established as in [35]. In the present paper, we obtain the real-time conditions on the augmented state by construction all the possible values of the augmented state, which is also suitable for [35] when "Assumption 9" is not applied and we think is one of the major difficulties overcome for extending the synthesis approach of MPC to network-induced delay environment in this paper. Further, there is an unavoidable fact in [36,35] that the online synthesis approach of MPC involves heavy computational burden which may prevent its practical application. As comparison, this paper provides an off-line synthesis approach of MPC by generalizing the procedure in [29]. The off-line synthesis approach of MPC makes the computational burden significantly reduced, which is preferable for such a practical NCS and we think could be useful for readers.

Notation: I is the identity matrix with appropriate dimension. For any vector x and matrix W, $\|x\|_W^2 := x^{\mathrm{T}}Wx$. The superscript T denotes the transpose for vectors or matrices. $x(k+i|k)$ is the value of vector x at a future time $k+i$ predicted at time k. The symbol $(*)$ is used to induce symmetric matrices.

2. Modeling of NCS with network-induced delays

The structure of NCS is depicted in Fig. 1, the plant is

$$x(k+1) = Ax(k) + Bu(k), \quad k \ge 0 \tag{1}$$

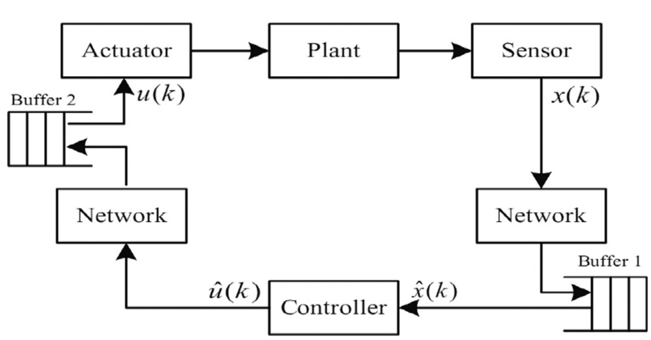

Fig. 1. Networked control systems with network-induced delays.

where A and B are constant matrices of appropriate dimensions; $u(k) \in \Re^m$ and $x(k) \in \Re^n$ are input and state of the plant, respectively; $\hat{u}(k) \in \Re^m$ and $\hat{x}(k) \in \Re^n$ are output and input of the controller, respectively. The assumptions for NCS are

- A1 The sensor is clock driven, i.e., it sends the sensed state $x(k)$ at each k;
- A2 The controller is event driven, i.e., at time k, the controller reads $\hat{x}(k)$ and calculates $\hat{u}(k)$ if and only if buffer 1 receives data;
- A3 The actuator is event driven, i.e., at time k, the actuator reads $u(k)$ and updates the input if and only if buffer 2 receives data;
- A4 In S–C link, the sensed state $x(k)$ is transmitted at each k, and the bounded network-induced delays may occur;
- A5 In C–A link, the control move $\hat{u}(k)$ is transmitted at each k, and the bounded network-induced delays may occur;
- A6 If the actuator does not receive any data, the input $u(k-1)$ is utilized;
- A7 The sensed state and control signal are marked with time-stamp.

2.1. Data transmissions in S–C link

In S–C link, since the network-induced delay is bounded and may be larger than one sampling period, three usual cases may occur at time k:

(a) Only one data packet arrives at buffer 1;
(b) More than one data packet arrives at buffer 1;
(c) There is no data packet arrives at buffer 1.

In case (a), the controller reads the arrived data and calculates the control move. In case (b), the controller reads the newest data for calculation, while the others are discarded. In case (c), the controller does not calculate. It is shown that the discarded sensed states do not affect the controller. The following time sequences and steps are introduced to describe the data transmission process in S–C link and identify the sensed states which affect the controller.

(1) Assume that $x(k)$ arrives at buffer 1 at time $\bar{j}_k$, where $\bar{j}_k$ is a non-negative integer. Then, the time points $\{\bar{j}_1, \bar{j}_2, \bar{j}_3, \ldots\}$ are assembled to form time sequence $\mathcal{J}_1$.
(2) For any three adjacent time points $\{\bar{j}_a, \bar{j}_{a+1}, \bar{j}_{a+2}\} \in \mathcal{J}_1$, if $\bar{j}_{a+1} - \bar{j}_a \ge 0$ and $\bar{j}_{a+2} - \bar{j}_{a+1} \ge 1$, then $\bar{j}_{a+1}$'s form the sequence $\mathcal{J}_2 := \{j_1, j_2, j_3, \ldots\}$ which affects the closed-loop system. Specially, if $\bar{j}_2 - \bar{j}_1 \ge 1$, then $\bar{j}_1 \in \mathcal{J}_2$.

For convenience, define k_{j_l} as the index such that $\hat{x}(j_l)$ is $x(k_{j_l})$.

Example 1. Fig. 2 shows an example of data transmissions from the sensor to the actuator. Different time sequences are illustrated as follows. First, it is easy to show that $\mathcal{J}_1 = \{1, 2, 2, 5, 6, 6, 7, 8, 9, 9, 11, 11, \ldots\}$. Second, by deleting $x(1), x(4), x(8), x(10)$, which are not read by the controller, $\mathcal{J}_2 = \{1, 2, 5, 6, 7, 8, 9, 11, \ldots\}$ is obtained.

2.2. Data transmissions in C–A link

The data transmission process in C–A link is similar to that in S–C link. When only one data packet arrives at buffer 2 during one sampling interval, the arrived data packet is utilized to update the control input; When more than one data packet arrives at buffer 2 during one sampling interval, the actuator only reads the most recent data for utilization; When no new data packet arrives buffer 2 during one sampling interval, the previous control input acts on the plant. Hence, only a part of control moves affect the actuator.