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Improved delay-range-dependent stability analysis of a time-delay system with norm bounded uncertainty

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ABSTRACT

This paper presents improved robust delay-range-dependent stability analysis of an uncertain linear time-delay system following two different existing approaches – (i) non-delay partitioning (NDP) and (ii) delay partitioning (DP). The derived criterion (for both the approaches) proposes judicious use of integral inequality to approximate the uncertain limits of integration arising out of the time-derivative of Lyapunov–Krasovskii (LK) functionals to obtain less conservative results. Further, the present work compares both the approaches in terms of relative merits as well as highlights tradeoff for achieving higher delay bound and (or) reducing number of decision variables without losing conservatism in delay bound results. The analysis and discussion presented in the paper are validated by considering relevant numerical examples.

1. Introduction

One usually assumes that the future state of the dynamical system is determined solely by the present state of the system and is independent of the past state information, whereas the class of system that includes past state information along with present state for finding the future state is referred as time-delay system (TDS) [1–4]. The ubiquitous presence of time-delay in any physical system (e.g., chemical processes, biological processes, process control, population dynamics and aerospace engineering) is known to be a source of instability and performance degradation of the system [1,5,4]. Assessment of stability in TDS involves computing the delay bound up to which system can retain stability, in sequel there are two ways for stability assessment for such systems – (i) time-domain technique and (ii) frequency-domain technique. The former technique has relative merit of ease in controller synthesis and computational ease although it yields conservative delay bound result, but the latter method can compute exact delay bound but due to computational complexity controller synthesis is difficult. The time-domain technique is adopted in this work for stability assessment as there is still room for increasing the delay bound. The method is based on Lyapunov's second method referred as L–K functional approach which subsequently formulates the stability condition in an LMI frame work. For a physical system it is natural to expect that the system will lose stability at a certain finite delay value, thus the derived stability condition contains the information of delay size in it – such conditions are referred in the literature as delay-dependent stability condition. Hence, vast research literature on stability analysis for TDS is directed towards deriving delay-dependent one because of its physical significance and can be found in [6,5] and references therein. Continuous improvement in the delay-dependent stability results are reported where attempts are made to reduce the conservativeness in the estimate of delay upper bound compared to the existing methods and can be found in [5] and references there in. Recently in [7–10] another notion of stability for such a system has been coined called – delay-range dependent stability. According to this notion, the delay lower bound is not restricted to zero but it is a small positive number thus giving the measure of delay range (difference between upper and lower delay values).

Delay-dependent or delay-range-dependent stability conditions are derived using L–K functional approach by two popular techniques – (i) by partitioning the delay range, which is referred here as delay partitioning (DP) approach [11–13] and (ii) by not partitioning the delay range, which is referred here as non-delay-partitioning (NDP) method [14–24]. The literature shows that partitioning a delay interval (DP approach) or adopting augmented LK functional involves more free matrices, whereas in NDP approach as no sub division of delay interval is carried out it

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involves a lesser number of free matrix variables compared to the former case. The relative merit of the DP over NDP approach is that the former method yields less conservative delay range compared to the latter one due to the involvement of lesser decision (matrix) variables. In this context, authors are of the view that it is improper to compare stability or robust stability results that follow different stability analysis techniques. In sequel, an attempt is made in this paper to bring out a comparison between the various results corresponding to DP or NDP methods.

Most of the literature on delay-range-dependent stability analysis are for nominal TDS, however few stability results on delay-range-dependent stability condition for uncertain time-delay system exist. Further, one can find robust delay-range-dependent or delay-dependent results following NDP methods in [13,14,17,18,23,25] and DP method in [7,15,16]. In [17] delay-dependent robust stability analysis of time-delay linear system with constant delay nature has been considered, [14] adopts robust stability analysis for such a system using augmented LK functional approach with much higher number of free variables, [18] considers stability problem for an uncertain system with polytopic parametric uncertainties. These above discussed notions of stability for time-delay system are of practical significance, like, stability assessment of networked controlled systems (NCS) [11,14,26], neural-network with time-delays [25,27–30], load-frequency control problem of an interconnected power system [31], biomedical applications [32] and many more problems from science and engineering.

The paper is organized as follows, Section 2 presents the problem statement along with few useful lemmas to derive the sufficient stability condition. Main delay-range-dependent robust stability results are presented in Section 3 using two different approaches (DP and NDP). Two benchmark numerical examples have been provided in Section 4 along with the results. Section 5 concludes the work.

Notations: The superscript T stands for transpose of a matrix, $\Re^n$ denote n dimensional Euclidean space. The notation '$*$' in a symmetric matrix denotes the symmetric terms and $diag\{\cdot\}$ stands for block diagonal matrix. The notation $Z>0$ (respectively $Z \ge 0$), for $Z \in \Re^n$ means that Z is real symmetric positive definite matrix (respectively, positive semi-definite matrix). $\mathcal{C}([-\overline{\tau},0],\Re^n)$ denotes the space of continuous differentiable function mapping from the interval $[-\overline{\tau},0]$ into $\Re^n$, and I is an identity matrix.

2. Problem statement

Consider an uncertain time-delay system described by

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t-\tau(t)), \tag{1}$$

$$x(t) = \phi(t), t \in [-\tau_2, 0] \tag{2}$$

where $x(t) \in \Re^n$ is the state vector, $\phi(t) \in \mathcal{C}([-\tau_2,0],\Re^n)$ is the initial condition of the system. The time-delay $\tau(t)$, in (1), is a time-varying continuous function satisfying the following conditions:

$$0 < \tau_1 \le \tau(t) \le \tau_2, \quad \overline{\tau} = \tau_2 - \tau_1, \quad 0 \le \dot{\tau}(t) \le \mu, \quad \forall t \ge 0; \tag{3}$$

where τ_1, τ_2, $\overline{\tau}$ and μ are constants and indicate delay lower bound, delay upper bound, delay range and delay-derivative upper bound respectively. In (1), $A(t)$ and $A_d(t)$ are uncertain system matrices that can be decomposed as

$$A(t) = A + \Delta A(t) \tag{4}$$

$$A_d(t) = A_d + \Delta A_d(t) \tag{5}$$

where $\Delta A(t)$ and $\Delta A_d(t)$ are time-varying matrices with norm-bounded parametric uncertain structure added to A and A_d nominal system matrices, respectively. The uncertain matrices may be further decomposed by exploiting their structural description as

$$[\Delta A(t) \;\; \Delta A_d(t)] = DF(t)[E_a \;\; E_d] \tag{6}$$

where D, E_a and E_d are known constant matrices that influence the parameter of the nominal system matrices A and A_d and $F(t)$ is an uncertain matrix satisfying

$$F^T(t)F(t) \le I \quad \text{or} \quad \| F(t) \| < 1. \tag{7}$$

Remark 1. The uncertain matrix $F(t)$ in (6) may be eliminated using the following lemma, such that the stability criterion is formulated in the form of LMI.

Lemma 2.1 (Liu [19]). *Given real matrices $\Omega = \Omega^T$, Ξ, Λ and uncertain matrix $F(t)$ of appropriate dimensions with $\|F(t)\| \le 1$, then for any scalar $\epsilon > 0$, the inequality*

$$0 > \Omega + \Xi F(t)\Lambda + \Lambda^T F^T(t)\Xi^T \tag{8}$$

can be equivalently written as

$$\begin{bmatrix} \Omega & \Xi & \epsilon\Lambda^T \\ \Xi^T & -\epsilon I & 0 \\ \epsilon\Lambda & 0 & -\epsilon I \end{bmatrix} < 0 \tag{9}$$

Remark 2. The bounding of the integral terms arising out of L–K functional derivative will be approximated using free matrix approach [33] as

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta)R\dot{x}(\theta)\,d\theta \le \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \left\{ \begin{bmatrix} M+M^T & -M+N^T \\ * & -N-N^T \end{bmatrix} + \gamma \begin{bmatrix} M \\ in \end{bmatrix} R^{-1} \begin{bmatrix} M \\ in \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}, \tag{10}$$

where '$*$' represents symmetric components, $R = R^T > 0$, $\beta > \alpha \ge 0$, $\gamma = \beta - \alpha > 0$ and M, N are free weighting matrices of appropriate dimension. However, in [8], it has been shown that use of such free weighting matrices may impose constraint on the resulting stability criterion and obtain less conservative results by using an integral type inequality (Jensens's inequality) of [34] given by

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta)R\dot{x}(\theta)\,d\theta \le \gamma^{-1} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}. \tag{11}$$

Many attempts have been made to deduce equivalency and conservativeness of several stability criteria based on either (10) or (11), e.g. see [35,8]. Explicit relation between (10) and (11) can be established following the equivalency results in [36]. In this regard, note that, the first term in the right hand side (RHS) of (10) may be represented as

$$\begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} I \\ -I \end{bmatrix}^T + \begin{bmatrix} I \\ -I \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix}^T. \tag{12}$$

Now, it is easy to see that (10) and (11) are equivalent in view of Theorem 4.1 of [36]. Moreover, the RHS of (10) is minimum when

$$M = M^T = -N = -N^T = -\gamma^{-1}R, \tag{13}$$

and for such a choice (10) becomes (11).

From the above, it seems that use of (11) is always desired since it does not involve additional free variables besides being equivalent to (10). However, if γ is uncertain and required to be approximated with its lower or upper bound then use of (10) would be beneficial since the choice (13) cannot be met with an approximated γ. Moreover, the RHS of (10) is affine on the