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Ultra-fast formation control of high-order discrete-time multi-agent systems based on multi-step predictive mechanism

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ABSTRACT

This paper deals with the ultra-fast formation control problem of high-order discrete-time multi-agent systems. Using the local neighbor-error knowledge, a novel ultra-fast protocol with multi-step predictive information and self-feedback term is proposed. The asymptotic convergence factor is improved by a power of $q+1$ compared to the routine protocol. To some extent, the ultra-fast algorithm overcomes the influence of communication topology to the convergence speed. Furthermore, some sufficient conditions are given herein. The ones decouple the design of the synchronizing gains from the detailed graph properties, and explicitly reveal how the agent dynamic and the communication graph jointly affect the ultra-fast formationability. Finally, some simulations are worked out to illustrate the effectiveness of our theoretical results.

1. Introduction

Multi-agent systems usually consist of a group of agents cooperating to complete certain tasks, and their coordination control has generated considerable research interest [1]. In this paper, we study the formation control problem for a group of agents, which has attracted much attention due to the potential applications, such as satellite attitude control, unmanned aircraft formation flying, distributed sensor networks, and automated highway systems (AHS) [2].

Compared to the traditional monolithic systems, the formation control reduces the systems cost, breaches the size constraints, and prolongs the life span of the systems [3]. Furthermore, the robustness and flexibility are enhanced. In [1,4], the authors deal with formation control problems for multi-agent systems by using iterative learning control design approaches. The authors of [5] formulate and study the distributed formation problem of multi-agent systems, and a formation controller is designed in a general form on the basis of artificial potential functions. A novel formation control strategy based on inter-agent distances for single-integrator modeled agents in the plane is proposed in [6]. The authors of [7] study the problem of formation control and trajectory tracking for a group of robotic systems modeled by Lagrangian dynamics. In conclusion, it is not difficult to see that, most of the existing work on formation control has been only focus on how to design the asymptotical control laws to stabilize the desired relative equilibrium, while lacked consideration of the convergence speed toward formationability. However, in many practical applications, the formation control algorithms, which obtain the formation faster or in finite time, are more desirable, especially when the multi-maneuver is needed and a high precision control is required. In other words, the convergence rate or speed is an important index to test the performance of different kinds of formation control protocols. As some of the very few papers that solve the formation control problem in finite time [8–10] consider agents that are modeled by single or double integrators. Nevertheless, in some applications, agents of higher dynamical order are required if formationability of more than two variables is aimed at. Therefore, the topic of this paper is that designing a new formation control protocol to solve the ultra-fast formationability for high-order discrete-time multi-agent systems. It is worth noting that the ultra-fast formationability means that the control protocols can incredibly fast drive all agents close to desired formation.

Most previous works on the formation control for multi-agent systems have been based on the implicit assumption that the available information at the next discrete-time step is solely determined by the current information. However, in natural bio-groups, individuals typically have some higher-level intelligence, namely predictive intelligence, which is the ability of predicting the future information of some group members based on their past and current information [11,12]. Motivated by the above analysis, using the local neighbor-

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error knowledge, a multi-step regulation-error predictive mechanism is established. By predicting the dynamics of a network several steps ahead and adding this information into the control strategy, a novel ultra-fast formation control protocol with self-feedback term is proposed. The ultra-fast protocol consists of three parts: actual neighbor-error, desired neighbor-error and self-feedback term. It can incredibly fast reduce the gap between the actual neighbor-error and desired neighbor-error. By comparing it with the routine protocol, it is shown that drastic improvements can be achieved in terms of the convergence speed. Specifically, the asymptotic convergence factor is improved by a power of $q+1$ compared to the routine protocol. The bigger the value for q is, the faster the convergence speed is. To some extent, the ultra-fast algorithm overcomes the influence of communication topology to the convergence speed. Furthermore, some sufficient conditions for ultra-fast formationability design are given herein. The ones decouple the design of the synchronizing gains from the detailed graph properties, and explicitly reveal how the agent dynamic and the communication graph jointly affect ultra-fast formationability of high-order discrete-time multi-agent systems. Finally, some simulations are worked out to illustrate the effectiveness of our theoretical results.

The rest of this paper is organized as follows. Section 2 introduces the concepts of communication graphs and problem formulation. Analysis and design on ultra-fast formationability are proceeded in Section 3. Illustrative examples are performed in Section 4. The conclusion remarks are drawn in Section 5.

Notation: Let Z, R^+, R, $\mathbb{C}$, $R^{m\times n}$ and $C^{m\times n}$ be the sets of integral number, positive real numbers, real numbers, complex numbers, real matrices and complex matrices, respectively. $|\mathbb{C}|$ denotes size of complex number $\mathbb{C}$. Given a matrix A, $\rho(A)$ is the spectral radius of its eigenvalue. The transpose (or conjugate transpose) of matrix A is denoted by A^T (or A^H). The inverse and Moore–Penrose inverse of matrix A are denoted by A^{-1} and A^+, respectively. The product of n matrix A is denoted by A^n. I_n is the identity matrix with dimension $n\times n$. $\mathbf{1}_n$ and $\mathbf{0}_n$ denote the $n\times 1$ column vectors whose elements are all ones and all zeros. $\|\cdot\|$ represents the standard ℓ^2 norm on vectors or their induced norms on matrices. diag $(A_1,\dots,A_N)$ is a block diagonal matrix with main diagonal block matrix A_j and zero off-diagonal block matrices. The Kronecker product, denoted by $\otimes$, facilitates the manipulation of matrices by the following properties: (1) $(A\otimes B)(C\otimes D)=AC\otimes BD$; (2) $(A\otimes B)^T=A^T\otimes B^T$; (3) $A\otimes B+A\otimes C=A\otimes(B+C)$.

2. Preliminaries on graph theory and problem formulation

2.1. Preliminaries on graph theory

Consider a graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ with a nonempty finite set of N vertices $\mathcal{V}=\{v_1,\dots,v_N\}$ and a set of edges or arcs $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. It is assumed that the graph is simple, i.e. there are no repeated edges or self-loops $(v_i,v_i)\in\mathcal{E},\forall i$. General directed graphs (digraphs) are considered and it is taken that information propagates through the graph along directed arcs. Denote the connectivity matrix as $\mathcal{A}=[\alpha_{ij}]\in R^{N\times N}$ with $\alpha_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$ and $\alpha_{ij}=0$ otherwise. Note that diagonal elements $\alpha_{ii}=0$. The set of neighbors of node v_i is denoted as $N_i\triangleq\{v_j|(v_j,v_i)\in\mathcal{E}\}$, i.e. the set of nodes with arcs incoming into v_i. Define the in-degree matrix as a diagonal matrix $\mathcal{D}\triangleq\text{diag}(d_1,\dots,d_N)$ with $d_i=\sum_j\alpha_{ij}$. Define the graph Laplacian matrix as $\mathcal{L}_\mathcal{G}=\mathcal{D}-\mathcal{A}$ and its eigenvalues in an ascending order are written as $0=\lambda_1(\mathcal{L}_\mathcal{G})\le\lambda_2(\mathcal{L}_\mathcal{G})\le\cdots\le\lambda_N(\mathcal{L}_\mathcal{G})$. For simplicity, $\lambda_j=\lambda_j(\mathcal{L}_\mathcal{G})$ will be used.

A directed path from node v_{i_1} to node v_{i_k} is a sequence of edges $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),\dots,(v_{i_{k-1}},v_{i_k})$, with $(v_{i_{j-1}},v_{i_j})\in\mathcal{E}$ for $j=2,\dots,k$. The graph is said to contain a (directed) spanning tree if there exists a vertex such that every other vertex in $\mathcal{V}$ can be connected by a directed path starting from it. Such a special vertex is then called a root. Note that for an undirected graph $\mathcal{G}$, $\mathcal{L}$ is a symmetric matrix.

Lemma 1 (*Royle and Godsil [13]*). *All the eigenvalues of $\mathcal{L}$ have nonnegative real parts. Zero is an eigenvalue of $\mathcal{L}$, with $\mathbf{1}_N$ as the corresponding right eigenvector.*

Lemma 2 (*Lin and Francis [14]*). *Zero is a simple eigenvalue of $\mathcal{L}$ if and only if graph $\mathcal{G}$ has a spanning tree.*

2.2. Problem formulation

In this paper, we study the ultra-fast formation control problem for high-order discrete-time multi-agent systems. The system to be considered consists of N autonomous agents, agent i is assumed to have the following dynamics:

$$x_i(k+1)=Ax_i(k)+Bu_i(k)\quad\forall i\in\mathcal{V},\quad k=0,1,\dots\tag{1}$$

where $x_i(k)\in R^{n\times 1}$ and $u_i(k)\in R^{m\times 1}$ respectively represent the state, control input of agent i. $A\in R^{n\times n}$ and $B\in R^{n\times m}$ are the state and input matrices, respectively. A desired formation vector $F=[F_1^T,F_2^T,\dots,F_N^T]^T,F_i\in R^{n\times 1}$ is given. It is well-known that the formationability can be achieved if and only if

$$\lim_{k\to\infty}\|[x_i(k)-F_i]-[x_j(k)-F_j]\|=0,\quad\forall i,j\in\mathcal{V}\tag{2}$$

Denote the coupling gain and control gain by $c\in R^+$ and $K\in R^{m\times n}$, respectively. By predicting the dynamics of a network several steps ahead and adding this information into the control strategy, a novel ultra-fast formation control protocol with the self-feedback term is proposed as follows:

$$u_i(k)=cK\sum_{h=0}^{q}[\hat{\varepsilon}_i(k+h)-f_i]+\Delta_i(k)\tag{3}$$

Obviously, we can see that protocol (3) consists of three parts:

(i) Actual neighbor-error (conclude predictive neighbor-error)

$$\hat{\varepsilon}_i(k+h)=\sum_{j\in N_i}\alpha_{ij}[\hat{x}_j(k+h)-\hat{x}_i(k+h)]\tag{4}$$

(ii) Desired neighbor-error

$$f_i=\sum_{j\in N_i}\alpha_{ij}(F_j-F_i)\tag{5}$$

(iii) Self-feedback information (conclude predictive information)

$$\Delta_i(k)=K'\sum_{h=1}^{q}\hat{x}_i(k+h)\tag{6}$$

where K' is self-feedback gain which satisfies $BK'=A-I_n$. To guarantee the existence of the value for K', we assume that $Rank(B)=Rank(B,A-I_n)$. Obviously, the self-feedback gain can be easily obtained by solving the above linear matrix equation. Further, an important assumption is given as follows:

Assumption 1. Agent i cannot only use the information relative to those of its neighboring agents, i.e. $\varepsilon_i(k)$, but also use its own information $x_i(k)$. Moreover, the global topology and desired formation, i.e. matrices L and F, are known for each agent, where $i=1,\dots,N$, $\varepsilon_i(k)=\sum_{j\in N_i}\alpha_{ij}[x_j(k)-x_i(k)]$ is called actual neighbor-error, and it is the available information for each agent.

Differ from the other works, in this paper, agent i considers the self-feedback information $x_i(k)$ that makes possible to achieve ultra-fast formation control. In addition, the corresponding