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Fault detection for discrete-time switched systems with sensor stuck faults and servo inputs

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ABSTRACT

This paper addresses the fault detection problem of switched systems with servo inputs and sensor stuck faults. The attention is focused on designing a switching law and its associated fault detection filters (FDFs). The proposed switching law uses only the current states of FDFs, which guarantees the residuals are sensitive to the servo inputs with known frequency ranges in faulty cases and robust against them in fault-free case. Thus, the arbitrarily small sensor stuck faults, including outage faults can be detected in finite-frequency domain. The levels of sensitivity and robustness are measured in terms of the finite-frequency H_- index and l_2-gain. Finally, the switching law and FDFs are obtained by the solution of a convex optimization problem.

1. Introduction

In recent years, the issue of fault detection (FD) has attracted amount of attention due to the increasing demand of safety and reliability standards in engineering practice [1–4]. Among the existing FD approaches, the model-based FD can replace hardware redundancy based on traditional schemes, which is widely applied for vehicle control systems, robots, power systems, manufacturing processes, see e.g. [5–8]. The basic idea of the mentioned subject is to design a detection unit to generate a signal named residual, which reflects the difference of system between faulty and fault-free cases. To this end, various types of the residual generator are proposed. For example, in [9], observer-based generator is developed to ensure detection performance by minimizing the errors between faults and residuals. To make the residuals be sensitive to faults and robust against disturbances, a mixed H_∞/H_- fault detection filter (FDF) is constructed in [10]. When the faults have known frequency domains, an improved frequency-based FDF is designed to reduce the conservatism [11].

On the other hand, switched systems have attracted increasing interest in the past three decades [12–14]. The systems appear in variety of applications, such as dc/dc convertors, FET transistors and HiMAT vehicle [15–17]. A switched system consists of a finite number of subsystems and a switching law orchestrating switching between these subsystems [18]. Lots of techniques have been developed to study the problems of stability and control for switched systems. For instance, [19] investigates the H_∞ control problem using piecewise quadratic Lyapunov function. In [20], the issue of quadratic stability is studied based on the multiple Lyapunov-like function. In the proposed technical frameworks, some typical switching laws which depend on time, state or both are constructed.

For the FD problem for switched systems, main objective is to design a switched law and residual generators. As is well known, a switched system can have good performance by properly constructing switching law. However, most relevant results pertain to time-dependent [21,22] or arbitrary switching [23]. Under the proposed switching laws, some FD performance in full-frequency domain are guaranteed. Recently, frequency-based FD approach is presented, which guarantees better FD performance [24]. Thus, it is worthwhile to consider how to design a novel switching law to satisfy finite-frequency FD performance of switched systems. On the other hand, sensor stuck fault is very common for control systems, which directly acts on the process measurement [25–27]. However, the fault type is not adequately addressed for switched systems. In the mentioned technical frameworks, the effect of sensor faults on residuals is directly considered. Clearly, when

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stuck faults have very small magnitudes, the fault sensitivity will be degraded. Even when the outage faults occur (stuck value is zero), it may fail to detect faults. Hence, another motivation of this paper arises: to detect arbitrarily small stuck faults including outage faults, can we present a novel FD scheme to remove the requirement of considering the relation between residuals and small sensor faults directly? Noting that the presence of servo inputs improves the performance of systems [28], we will solve the considered problem via introducing servo inputs.

This paper studies the FD problem of switched systems with servo inputs and sensor stuck faults. The contributions lie in the following aspects: (i) Different from the previous results, a state-dependent switching law is constructed, which uses only the states of FDFs. (ii) Under the switching law, the finite-frequency H_- index and l_2-gain are introduced to guarantee better fault sensitivity for the system. (iii) By constructing the difference between the residuals and servo inputs in fault-free and faulty cases, arbitrarily small sensor stuck faults including outage faults can be detected. This overcomes the drawback caused by considering the relation between the residuals and sensor stuck faults. Finally, the switching law and FDFs are obtained by solving linear matrix inequalities (LMIs).

The rest of the paper is organized as follows. In Section 2, some fundamental definitions, lemmas and main objectives are provided. The main results are proposed in Section 3. Section 4 gives the FD scheme. In Section 5, simulation examples are given to illustrate the effectiveness of the proposed method. Finally, some conclusions end the paper in Section 6.

The notation used in this paper is fairly standard. For a matrix A, A^T denotes its transpose. The Hermitian part of a square matrix M is denoted by $He(M)=M+M^T$. The symbol $*$ within a matrix represents the symmetric entries. 0 and I represent the zero and identity matrix with the appropriate dimensions, respectively.

2. Problem formulation and preliminaries

2.1. System model

Consider the following switched system:

$$\begin{aligned} x(k+1) &= A_\sigma x(k)+B_{d\sigma}d(k)+B_{v\sigma}v_\sigma(k) \\ y(k) &= C_\sigma x(k)+D_{d\sigma}d(k)+D_{v\sigma}v_\sigma(k) \end{aligned} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state, $y(k)\in\mathbb{R}^m$ is the measured output, $d(k)\in\mathbb{R}^r$ is the unknown input, and $v_\sigma(k)\in\mathbb{R}^v$ is the servo input with known frequency range. The unknown input and the servo input belong to $l_2[0,\infty)$. The piecewise constant function $\sigma(k):[0,\infty)\to\mathcal{L}=\{1,\ldots,N\}$ is a switching signal, that is, when $\sigma(k)=i$, the ith subsystem is activated, where $\mathcal{L}$ is a finite set and the positive integer N is the number of subsystems. The matrices A_i, B_{di}, B_{vi}, C_i, D_{di} and D_{vi} are known constant matrices with the appropriate dimensions.

2.2. Fault model

In this paper, we consider the following sensor stuck fault models:

$$y^{F_\sigma^p}(k)=F_\sigma^p y(k)+(I-F_\sigma^p)f_\sigma^p,\quad p=0,1,\ldots,q \tag{2}$$

where $f_\sigma^p=\left[f_{1\sigma}^p\ f_{2\sigma}^p\ \ldots\ f_{m\sigma}^p\right]^T$, $f_{l\sigma}^p$ $(l=1,\ldots,m)$ is the unknown constant, which represents the stuck value of the lth sensor. p denotes the pth fault mode, q is the number of the total possible fault modes and F_σ^p is the diagonal matrix with the following form:

$$F_\sigma^p=\operatorname{diag}\left[F_{1\sigma}^p\ F_{2\sigma}^p\ \cdots\ F_{m\sigma}^p\right],\quad F_{l\sigma}^p=0 \text{ or } 1 \tag{3}$$

Then, for $\sigma=i$, we know that, when $F_{li}^p=0$, $p=1,\ldots,q$, the lth sensor gets stuck, when $F_{li}^p=1$, $p=1,\ldots,q$, the lth sensor is fault-free and when $F_{li}^p=1$, $p=0$, $l=1,\ldots,m$, all the sensors are fault-free, that is, $y^{F_\sigma^0}(k)=y(k)$.

Remark 1. By revisiting the existing FD method in [25], the stuck value f_{li}^p is required to be known, which enhance the sensitivity between residuals and sensor stuck faults. Hence, the method cannot be applied when f_{li}^p is unknown. In this paper, the sensor faults with unknown stuck values can be detected by introducing servo inputs. This removes the restriction of the existing results.

2.3. Fault detection filters with servo inputs

For each subsystem i, the following FDF with servo input is constructed:

$$\begin{aligned} \hat{x}(k+1) &= A_{fi}\hat{x}(k)+B_{fi}y^{F_i^p}(k)+B_{fvi}v_i(k) \\ r_p(k) &= C_{fi}\hat{x}(k)+D_{fi}y^{F_i^p}(k) \end{aligned} \tag{4}$$

where $\hat{x}(k)\in\mathbb{R}^n$ is the state of the filter and $r_p(k)\in\mathbb{R}^s$ is the residual. A_{fi}, B_{fi}, B_{fvi}, C_{fi} and D_{fi} are filter matrices with appropriate dimensions, which are to be determined.

Remark 2. Letting $B_{fvi}=0$ or $v_i(k)=0$, the proposed FDF is converted into the existing one without servo input [25,26]. On the other hand, it has been proven that frequency-based filter leads to less conservatism [6,24]. Thus, by constructing $v_i(k)$ in finite-frequency domain, the switched system can have better FD performance. The frequency-based FDF design conditions will be provided in this paper.

Combining Eqs. (1)–(4), the overall system is described by

$$\begin{aligned} \tilde{x}(k+1) &= \overline{A}_i^p\tilde{x}(k)+\overline{B}_{di}^p d(k)+\overline{B}_{vi}^p v_i(k)+\overline{F}_{1i}^p f_i^p \\ r_p(k) &= \overline{C}_i^p\tilde{x}(k)+\overline{D}_{di}^p d(k)+\overline{D}_{vi}^p v_i(k)+\overline{F}_{2i}^p f_i^p \end{aligned} \tag{5}$$

where

$$\tilde{x}(k)=\begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix},\quad \overline{A}_i^p=\begin{bmatrix} A_i & 0 \\ B_{fi}F_i^pC_i & A_{fi} \end{bmatrix},\quad \overline{B}_{di}^p=\begin{bmatrix} B_{di} \\ B_{fi}F_i^pD_{di} \end{bmatrix},$$

$$\overline{B}_{vi}^p=\begin{bmatrix} B_{vi} \\ B_{fi}F_i^pD_{vi}+B_{fvi} \end{bmatrix},\quad \overline{F}_{1i}^p=\begin{bmatrix} 0 \\ B_{fi}(I-F_i^p) \end{bmatrix},\quad \overline{C}_i^p=\left[D_{fi}F_i^pC_i\ \ C_{fi}\right],$$

$$\overline{D}_{di}^p=D_{fi}F_i^pD_{di},\quad \overline{D}_{vi}^p=D_{fi}F_i^pD_{vi},\quad \overline{F}_{2i}^p=D_{fi}(I-F_i^p)$$

2.4. Preliminaries

The following definitions are given, which are essential for our derivation.

Definition 1. For each senor fault mode p, switched system (5) under switching law σ is said to have a finite-frequency H_- index γ_1, if the following inequalities

$$\sum_{k=0}^{\infty} r_p^T(k)r_p(k)\geq\gamma_1^2\sum_{k=0}^{\infty} v_i^T(k)v_i(k),\quad p=1,\ldots,q \tag{6}$$

hold for all solutions of (5) such that

$$\sum_{k=0}^{\infty}(\tilde{x}(k+1)-\tilde{x}(k))(\tilde{x}(k+1)-\tilde{x}(k))^T\leq\left(2\ \sin\frac{\theta_{li}}{2}\right)^2\sum_{k=0}^{\infty}\tilde{x}(k)\tilde{x}^T(k) \tag{7}$$