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Guaranteed cost control of mobile sensor networks with Markov switching topologies

Yuan Zhao, Ge Guo*, Lei Ding

School of Information Sciences and Technology, Dalian Maritime University, Linghai Road 1#, Dalian 116026, China

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ABSTRACT

This paper investigates the consensus seeking problem of mobile sensor networks (MSNs) with random switching topologies. The network communication topologies are composed of a set of directed graphs (or *digraph*) with a spanning tree. The switching of topologies is governed by a Markov chain. The consensus seeking problem is addressed by introducing a global topology-aware linear quadratic (LQ) cost as the performance measure. By state transformation, the consensus problem is transformed to the stabilization of a Markovian jump system with guaranteed cost. A sufficient condition for global mean-square consensus is derived in the context of stochastic stability analysis of Markovian jump systems. A computational algorithm is given to synchronously calculate both the sub-optimal consensus controller gains and the sub-minimum upper bound of the cost. The effectiveness of the proposed design method is illustrated by three numerical examples.

1. Introduction

In the past decade, wireless sensor networks have received a great deal of research attention due to their diverse applications in industrial automation, health monitoring, environment and climate monitoring, intruder detection, etc., [1]. In a dangerous or hostile environment, sensors cannot be manually deployed and fixed. It is necessary to deploy sensors mounted on mobile platforms such as unmanned vehicles, mobile robots, and spacecraft or man-made satellites. These sensors can collaborate among themselves to set up a sensing/actuating network, which is called a mobile sensor network (MSN). A typical MSN consists of hundreds or thousands of mobile sensor nodes distributed over a spatial region. Each sensor node has some level of capability for sensing, communication, signal processing and movement. The tendency that MSNs operated in a distributed manner will make use of small low power mobile devices may play revolutionary impact on many civil and military applications in exploration and monitoring.

Due to the limitation of resource, MSNs have limited costs for communication, computation and motion sub-capabilities. As a result, power-aware algorithms have recently been the subjects of extensive research [2–6] regarding various key issues such as localization, deployment, environment estimation and coverage control, rendezvous and consensus. For example, energy-efficient localization algorithms were proposed to reposition sensors in desired locations in order to recover or enhance network coverage or to maximize the covered area in [7,8] and [9]. In [8–10], the power-constrained deployment and coverage control issues were addressed by modeling energy consumption by the total traveling distance of the sensors. In [11] and [12], the vehicle speed management and the optimization problem of the number of agents for adequate coverage were addressed. In [13], a new algorithm for the maximum distance which an agent could travel by a dynamically changing energy radius was presented to solve the distributed deployment problem. An energy aware protocol which can prevent the agents from depleting their energy in achieving rendezvous was proposed in [14].

Consensus seeking, which means a group of mobile sensors achieve agreement upon a common state (i.e., position, velocity and direction), is another interesting problem in cooperative control of MSNs. There have been many papers studying consensus problems with cost optimization. To mention a few, an optimal consensus control method was proposed in [15] to minimize energy cost of sensors deployed in intelligent buildings for resource allocation. In [16], by introducing the cost functions to weigh both the consensus regulation performance and the control effort, an LQR consensus method was derived for multivehicle systems with single integrator dynamics. In [17], an optimal consensus seeking problem was studied in a network of general linear multi-agents. In [18], a two-step sub-optimal consensus control algorithm guaranteeing minimum energy cost for mobility and communication sub-tasks were derived.

* Corresponding author.
E-mail address: geguo@yeah.net (G. Guo).

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However, most of the above researches assume a static communication topology of MSNs. In practice, MSNs may have a dynamic network topology caused by link failure, packet dropout or environmental disturbances. [19] proposed a theoretical framework to study the consensus problem of multi-agent systems with switching topology. Based on nonnegative matrix theory, [20] and [21] investigated consensus control of multi-agent systems under dynamic topology. [22] considered a tradeoff between system performance and control effort of multi-agent systems with switching topologies. In some cases, due to random network conditions or environmental factors (e.g., sea wave, the wind and weather condition, [23]), an MSN may experience a randomly switching communication topology. Recently, increasing research attention has been paid to multi-agent systems with randomly switching network topologies, especially those with Markov switching topologies [24–27]. For example, [28] studied the almost sure convergence to consensus for agent network with Markovian switching topologies. By using the pth moment exponential stability theory and M-matrix approach, [29] considered the average consensus for the wireless sensor networks with Markovian switching topology and stochastic noise. In these results, it is required that Markov chains are ergodic, which implies that the multi-agent systems experience switching topologies in infinite time horizon. In other words, the systems cannot stay in a certain topology. In many practical applications, it is however more reasonable that systems may go though from switching topologies to a certain fixed topology. An example can be found when the systems pass from unsteady environment to a settled one. In fact, the control cost of a mobile sensor network depends on the communication and mobility behaviors of the sensors as well as the network topology. Therefore, for MSNs with Markov switching topologies, it is of great importance how to delicately involve the network topology factor into the control cost in setting up a low cost consensus control protocol. However, there are few results available on guaranteed cost control for consensus of multi-agent systems with Markov switching topologies.

In this paper, we aim to investigate the problem of guaranteed cost consensus seeking of MSNs with Markov switching topologies. We consider a collection of mobile sensors whose dynamics is described by a discrete-time state space equation. The communications topologies are assumed to be a set of directed graphs with a spanning tree. The switching of network topology is modeled as a Markov chain. A topology-dependent consensus protocol without local feedback is proposed, where the subtle structural dynamics of the switching topology is involved. A global LQ cost function depending on the control input and the state errors of neighboring sensors is introduced. Then, using graph theory and model transformation, the consensus problem with guaranteed cost is transformed to the problem of guaranteed cost stabilization of a reduced order Markov jumping system. A sufficient condition which guarantees global exponential consensus of the MSN in the mean square sense is derived based on stochastic Lyapunov functional method. A computational algorithm by which the consensus controller gains and a minimum upper bound of the cost can be calculated is given. The effectiveness of the consensus control method is illustrated by three numerical examples.

The remainder of this paper is organized as follows. Section 2 gives some preliminaries of graph theory and the problem formulation. Section 3 contains the main results on the sufficient condition of consensus and controller design for MSNs with Markov switching topology. Numerical examples are given in Section 4, which is followed by the conclusion in Section 5.

Notations: R^n denotes n dimensional Euclidean space, $R^{n\times m}$ represents the family of $n\times m$ dimensional real matrices. I_n is the identity matrix of dimension n. For a given vector or a matrix X, $X^T$and $||X||$ denotes its transpose and its Euclidean norm. $\rho(X)$ means the eigenvalue of matrix X. For a square nonsingular matrix X, X^{-1} denotes its inverse matrix. And $diag\{\dots\}$ stands for a block-diagonal matrix. For symmetric matrices P and Q, $P>Q$ (respectively, $P\geq Q$) means that matrix P–Q is positive define (positive semi-definite). The sign $\otimes$ represents matrix Kronecker product. $\mathbf{1}$ denotes a column vector whose entries equal to one. Similar notation is adopted for $\mathbf{0}$. The symmetric elements of a symmetric matrix are demoted by $*$. $\mathrm{E}(y)$ and $\mathrm{Pro}(y)$ are the mathematical expectation and probability of stochastic variable y. N^+ stands for non-negative integers.

2. Preliminaries and problem formulation

2.1. Preliminaries of graph theory

We use a directed graph (*digraph*) $G(\upsilon,\varepsilon,\Lambda)$ to model the interactions among sensors, where $\upsilon\in\{\upsilon_1,\cdots,\upsilon_N\}$ is the set of N nodes, $\varepsilon\subseteq\upsilon\times\upsilon$ is the set of edges, $\Lambda=[a_{ij}]$ is the adjacency matrix with its elements associated with the edges, i.e., if $\upsilon_i,\upsilon_j,\in\varepsilon$, $a_{ij}>0$, otherwise $(\upsilon_i,\upsilon_j)\notin\varepsilon$, a_{ij}=0. In the paper we will consider graphs without self-edge, i.e., $a_{ii}=0$. Each edge $(\upsilon_i,\upsilon_j)\in\varepsilon$ implies that node υ_i can receive information from node υ_j. A sequence of edges (υ_i,υ_k), (υ_k,υ_l), … , (υ_m,υ_j) is called a directed path from node υ_j to node υ_i. A digraph is said to have a spanning tree, if there is a root (which has only children but no parent) such that there is a directed path from the root to any other nodes in the graph. The set of neighbors of node υ_i is denoted by $N_i=(\upsilon_j\in\upsilon:(\upsilon_i,\upsilon_j)\in\varepsilon)$. Define the in-degree of node υ_i as $d_i=\sum_{j=1}^N a_{ij}$ and in-degree matrix $\Delta=diag\{d_1,\cdots d_N\}$. The Laplacian matrix of the directed graph G is defined as $L=\Delta-\Lambda$. Accordingly, define the out-degree of node υ_i as $d_i{}^o=\sum_{j=1}^N a_{ji}$ and the out-degree matrix $\Delta^o=diag\{d_1{}^o,\cdots,d_N{}^o\}$. The graph column Laplacian matrix of the directed graph G is defined as $L^o=\Delta^o-\Lambda^T$. An important property of L is that all of its row sums are zero, thus $\mathbf{1}$ is an eigenvector of L associated with eigenvalue zero. Zero is a simple eigenvalue of L if and only if the directed graph has a spanning tree, and the other eigenvalues are with positive real parts.

2.2. Markov switching topology

Consider a mobile sensor network with N identical sensors. At every instant k, the interconnection of these sensors can be considered as a directed graph with a spanning tree. The communication topology is switching but not fixed due to a certain random event. Assume that the topology is switching within a given set of graphs $G(\theta(k))\in\overline{G}(k)$, $\overline{G}(k)=\{G_1,G_2,\cdots,G_q\}$, where $\{\theta(k),k\in N^+\}$ is the switching signal. Here, $\theta(k)\in S=\{1,\cdots,q\}$is assumed to be a Markov chain taking values in a finite set. Its transition probability is given as

$$\mathrm{Pro}\{\theta(k+1)=v|\theta(k)=l\}=\pi_{lv},$$

with

$$\mathrm{Pro}(\theta(k)=l)=\pi_l(k)$$

where$l,v\in S$, $\pi_l(k)$ is the transition probability of G_l at time k with initial probability $\mathrm{Pro}(\theta(0)=l)=\pi_{0l}$, and π_{lv} is the single step transition probability from mode l to mode v, which satisfies $\sum_{v=1}^{q}\pi_{lv}=1$. The adjacency matrix $\Lambda(\theta(k))$ and Laplacian matrix of graph $G(\theta(k))$ are defined as $\Lambda(\theta(k))\in\{\Lambda_1,\Lambda_2,\cdots\Lambda_q\}$ and $L(\theta(k))\in\{L_1,L_2,\cdots L_q\}$, respectively.

Denote the whole topology modal probability distribution by matrix $\Pi(k)=[\pi_1(k),\cdots,\pi_q(k)]^T$, with initial probability distribution $\Pi_0=[\pi_{01},\cdots,\pi_{0q}]^T$. Let $\pi=[\pi_{lv}]_{q\times q}$ be the transition probability matrix. Then we have $\Pi(k+1)=\pi^T\Pi(k)$.