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State estimation of stochastic non-linear hybrid dynamic system using an interacting multiple model algorithm

M. Elenchezhiyan, J. Prakash *

Department of Instrumentation Engineering, Madras Institute of Technology Campus, Anna University, Chennai 600044, India

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ABSTRACT

In this work, state estimation schemes for non-linear hybrid dynamic systems subjected to stochastic state disturbances and random errors in measurements using interacting multiple-model (IMM) algorithms are formulated. In order to compute both discrete modes and continuous state estimates of a hybrid dynamic system either an IMM extended Kalman filter (IMM-EKF) or an IMM based derivative-free Kalman filters is proposed in this study. The efficacy of the proposed IMM based state estimation schemes is demonstrated by conducting Monte-Carlo simulation studies on the two-tank hybrid system and switched non-isothermal continuous stirred tank reactor system. Extensive simulation studies reveal that the proposed IMM based state estimation schemes are able to generate fairly accurate continuous state estimates and discrete modes. In the presence and absence of sensor bias, the simulation studies reveal that the proposed IMM unscented Kalman filter (IMM-UKF) based simultaneous state and parameter estimation scheme outperforms multiple-model UKF (MM-UKF) based simultaneous state and parameter estimation scheme.

1. Introduction

Dynamic systems that are described by an interaction between continuous dynamics and discrete modes are called hybrid dynamic systems [12,13]. In practice, hybrid dynamic systems are affected by random disturbances and measurements are corrupted with random noise. Thus, state estimation of stochastic hybrid dynamic system poses a challenging problem [15].

The Kalman update based non-linear state estimators such as the extended Kalman filter (EKF) [10,27], the unscented Kalman filter (UKF) [11] and the ensemble Kalman filter [22,26] and Particle filter [25] and their variants have been widely used to estimate the state variables of chemical processes such as Distillation column, continuous stirred tank reactors, Polymerization reactors etc. [20]. In the context of developing state estimator for hybrid dynamic systems, [17] suggested that the derivative-free state estimation schemes appear to be promising candidates. Excellent review articles on state and parameter estimation schemes have appeared recently in the process control literature [19,20].

The use of moving horizon approach based state estimation for hybrid system is reported in [2,8,9]. The design of a non-linear state feedback control system for a class of switched non-linear system has been addressed in [7]. It may be noted that [7] have designed a deterministic high gain observer for the estimation of state variables. Robust state estimation and fault diagnosis for an uncertain hybrid dynamic system is reported in [23]. Recently, a state estimation scheme for a non-linear autonomous hybrid system, which is subjected to stochastic state disturbances and measurement noise, using the ensemble Kalman filter and unscented Kalman filter, has been proposed by [17]. Simultaneous estimation of both continuum and non-continuum state variables with an application to the distillation process using a moving horizon estimator is reported in [15,16]. Fault detection and monitoring scheme for a non-isothermal chemical reactor with uncertain mode transitions using an unknown input observer theory and results from Lyapunov stability theory is proposed recently by [24].

Many chemical processes are characterized by strong interactions between continuous dynamics and discrete events and are more appropriately modeled by hybrid systems [12,13]. On-line estimation of state variables of such systems is very important from the view point of fault detection and identification and model based control [4,6,14,24]. It may be noted that the interacting multiple-model algorithm is a well-established state estimation technique and being widely used to estimate the continuous states and discrete modes in the area of target tracking applications [1,3].

The major contributions of the work are as follows: a state estimation scheme for stochastic non-linear hybrid system which has continuous dynamics and discrete modes modeled by a finite Markov chain using either IMM-EKF (or) IMM-UKF (or) IMM based ensemble Kalman filter (IMM-EnKF) is proposed. An IMM based simultaneous state and parameter estimation scheme to estimate the state variables and discrete modes of hybrid dynamic system in the presence of

* Corresponding author.
E-mail address: prakaiit@gmail.com (J. Prakash).

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sensor bias is also proposed. The efficacy of the proposed state estimation schemes is validated by carrying out Monte-Carlo simulation studies on the simulated models of non-linear two-tank hybrid system and switched non-isothermal continuous stirred tank reactor. Also, the performance of the IMM-UKF based simultaneous state and parameter estimation scheme has been validated with the experimental data and is found to be consistent with the simulation results.

The organization of the paper is as follows: Section 2 describes the problem formulation and Section 3 deals with state estimation for hybrid dynamic system using IMM based state estimators. Extensive simulation studies have been reported in Section 4 and followed by concluding remarks in Section 5.

2. IMM based state estimation scheme for a hybrid dynamic system

In this work, it is proposed to formulate the state estimation problem associated with the sub-class of hybrid dynamic system represented by the following equations:

$$\mathbf{x}(k)=\mathbf{x}(k-1)+\left[\int_{(k-1)T}^{kT}F^{(i)}[\mathbf{x}(\tau),\mathbf{u}(k-1)]d\tau\right]+\mathbf{w}^{(i)}(k) \quad (1)$$

$$\mathbf{y}(k)=H[\mathbf{x}(k)]+\mathbf{v}(k) \quad (2)$$

In the above process model, $\mathbf{x}(k)$ is the system state vector ($\mathbf{x}\in R^n$), $\mathbf{u}(k)$ is the known system input ($\mathbf{u}\in R^m$), $\mathbf{w}^{(i)}(k)$ is the process noise ($\mathbf{w}^{(i)}\in R^n$) with known distribution, $\mathbf{y}(k)$ is the measured state variable ($\mathbf{y}\in R^r$) and $\mathbf{v}(k)$ is the measurement noise ($\mathbf{v}(k)\in R^r$) with known distribution. The index 'k' represents the sampling instant and the index 'i' $\in\{1,2....N\}$ represents the discrete mode whose evolution is governed by the finite state Markov chain as shown below

$$\boldsymbol{\mu}(k)=\Omega\boldsymbol{\mu}(k-1) \quad (3)$$

where $\Omega=\{\Omega_{ij}\}\in R^{N\times N}$ is the mode transition matrix and $\boldsymbol{\mu}(k)\in R^N$ is the mode probability at time 'k'. The non-linear process and measurement models for each mode $F^{(i)}$ and $H^{(i)}$ are assumed to be known in this work. In this work, it is assumed that the initial state vector follows multivariate normal distribution with known mean vector and covariance matrix and the noise sequences $\{\boldsymbol{w}^{(i)}(k)\}$ and $\{\mathbf{v}(k)\}$ are assumed to be zero mean, normally distributed and mutually uncorrelated white noise sequences with known covariance matrices namely $\mathbf{Q}^{(i)}$ and $\mathbf{R}$ respectively. For each mode, we recommend the use of either an extended Kalman filter or an unscented Kalman filter or an ensemble Kalman filter to estimate the continuous state variables of the hybrid dynamic system.

The steps involved in obtaining state estimates and discrete modes of hybrid dynamic system using the IMM approach are as follows.

The IMM algorithm consists of 'N' interacting Kalman update based non-linear filters (EKF or UKF or EnKF algorithms as outlined in Appendix A, B, and C respectively) operating in parallel. However, it should be noted that in the IMM approach, at discrete time 'k' the state estimate is computed under each model using 'N' Kalman update based non-linear filters, with each non-linear Kalman filter using a different combination of the previous model-conditioned estimates (mixed initial condition [1]). The one cycle of the IMM algorithm [1] consists of the following steps:

(i) Calculation of the mixing probabilities and mixed initial condition [1]
The probability that the mode $M^{(i)}$ was in effect at discrete time instant '$k-1$' given that $M^{(j)}$ is in effect at discrete time instant 'k' conditioned on $\mathbf{Y}^{k-1}$ is

$$\lambda_{i|j}(k-1|k-1)=\frac{\Omega_{ij}\boldsymbol{\mu}^{(i)}(k-1)}{\sum_{i=1}^{N}\Omega_{ij}\boldsymbol{\mu}^{(i)}(k-1)} \quad i,j=1,2,...N \quad (4)$$

where $\lambda_{i|j}(k-1|k-1)$ is mixing probability, Ω_{ij} is the assumed transition probability for the Markov chain according to which the system model switches from model 'i' to model 'j', $\boldsymbol{\mu}^{(i)}(k-1)$ is the mode probability at discrete time instant '$k-1$'. The input to each Kalman update based non-linear state estimator matched to model 'j' is obtained from an interaction of the 'N' Kalman update based non-linear state estimator which consists of the mixing of estimates $\hat{\mathbf{x}}^{(i)}(k-1|k-1)$ with the weights $\lambda_{i|j}(k-1|k-1)$, called the mixing probabilities

$$\overline{\mathbf{x}}^{(j)}(k-1|k-1)=\sum_{i=1}^{N}\hat{\mathbf{x}}^{(i)}(k-1|k-1)\lambda_{i|j}(k-1|k-1); \quad j=1...N \quad (5)$$

The error covariance matrix is computed as

$$\begin{aligned}\overline{\mathbf{P}}^{(j)}(k-1|k-1)=\sum_{i=1}^{N}&\lambda_{i|j}(k-1|k-1)[\mathbf{P}^{(i)}(k-1|k-1)\\&+[\hat{\mathbf{x}}^{(i)}(k-1|k-1)-\overline{\mathbf{x}}^{(j)}(k-1|k-1)][\hat{\mathbf{x}}^{(i)}(k-1|k-1)\\&-\overline{\mathbf{x}}^{(j)}(k-1|k-1)]^T; \quad j=1...N\end{aligned} \quad (6)$$

(ii) Mode-matched filtering [1]
The state estimate ($\overline{\mathbf{x}}^{(j)}(k-1|k-1)$) and the error covariance matrix at previous time instant ($\overline{\mathbf{P}}^{(j)}(k-1|\ k-1)$) are used as input to each Kalman update based non-linear filter (refer to Appendix A, B, and C) matched to $M^{(j)}$ which uses the current measurement $\mathbf{y}(k)$ to yield $\hat{\mathbf{x}}^{(j)}(k|k)$ and $P^{(j)}(k|k)$. The likelihood functions corresponding to the 'N' Kalman update based non-linear state estimators are computed as follows:

$$\boldsymbol{\Lambda}^{(j)}(k)=\frac{\exp\left[-0.5\left[\boldsymbol{\gamma}^{(j)}(k|k-1)^T\left[\mathbf{V}^{(j)}(k)\right]^{-1}\boldsymbol{\gamma}^{(j)}(k|k-1)\right]\right]}{\sqrt{(2\pi)^r|\mathbf{V}^{(j)}(k)|}} \quad (7)$$

In the above equation, $\boldsymbol{\gamma}^{(j)}(k|k-1)$, $\mathbf{V}^{(j)}(k)$ are the innovation and innovation covariance matrices respectively.

(iii) Mode probability update [1]
The mode probabilities are computed as follows:

$$\boldsymbol{\mu}^{(j)}(k)=P\left[M^{(j)}|\mathbf{Y}^k\right]=\frac{\boldsymbol{\Lambda}^{(j)}(k)\left[\sum_{i=1}^{N}\Omega_{ij}\boldsymbol{\mu}^{(i)}(k-1)\right]}{\sum_{l=1}^{N}\boldsymbol{\Lambda}^{(l)}(k)\left[\sum_{i=1}^{N}\Omega_{ij}\boldsymbol{\mu}^{(i)}(k-1)\right]}$$
$$j=1,2,...N \quad (8)$$

(iv) State estimate and covariance combination [1]
The updated state estimates and updated error covariance matrix are computed using the following equations:

$$\hat{\mathbf{x}}(k|k)=\sum_{j=1}^{N}\hat{\mathbf{x}}^{(j)}(k|k)\boldsymbol{\mu}^{(j)}(k) \quad (9)$$

$$\boldsymbol{P}(k|k)=\sum_{j=1}^{N}\boldsymbol{\mu}^{(j)}(k)\left[\boldsymbol{P}^{(j)}(k|k)+\left[\hat{\boldsymbol{x}}^{(j)}(k|k)-\hat{\boldsymbol{x}}(k|k)\right]\left[\hat{\boldsymbol{x}}^{(j)}(k|k)-\hat{\boldsymbol{x}}(k|k)\right]^T\right] \quad (10)$$

3. Simulation studies

The efficacy of the state estimation schemes namely IMM-EKF, IMM-UKF and IMM-EnKF has been validated on the two benchmark examples namely (i) two-tank hybrid system and (ii)