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Research Article

Robust fault detection for switched positive linear systems with time-varying delays

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ABSTRACT

This paper investigates the problem of robust fault detection for a class of switched positive linear systems with time-varying delays. The fault detection filter is used as the residual generator, in which the filter parameters are dependent on the system mode. Attention is focused on designing the positive filter such that, for model uncertainties, unknown inputs and the control inputs, the error between the residual and fault is minimized. The problem of robust fault detection is converted into a positive L_1 filtering problem. Subsequently, by constructing an appropriate multiple co-positive type Lyapunov–Krasovskii functional, as well as using the average dwell time approach, sufficient conditions for the solvability of this problem are established in terms of linear matrix inequalities (LMIs). Two illustrative examples are provided to show the effectiveness and applicability of the proposed results.

1. Introduction

A switched positive linear system (SPLS) is a type of hybrid dynamical system that consists of a number of positive subsystems [1,2] and a switching signal, which defines a specific positive subsystem being activated during a certain interval. SPLS deserves investigation both for practical applications and for theoretical reasons. Especially, the positivity constraint is pervasive in engineering practice as well as in chemical, biological and economic modeling, etc. See for instance, communication networks [3], the viral mutation dynamics [4], formation flying [5], and system theory [6–10]. It should be pointed out that studying SPLS is more challenging than that of general switched system or positive system because, in order to obtain some results, one has to combine the features of positive systems and switched systems. Recently, some results on the stability analysis of SPLSs have been obtained [11–13].

In practice, time-delay phenomena in dynamic systems widely exist. Although many results have been reported for time-delay systems [14–25], only recently has the SPLS with time delay become a topic of major interest [26–28], which is theoretically challenging and of fundamental importance to numerous applications.

On the other hand, fault detection and isolation (FDI) in dynamic systems has been an active field of research during the past decades, and some model-based fault detection approaches have been proposed in [29–38]. The basic idea of the model based FDI is to use state observers or filters to generate a residual signal and, based on this, to determine the residual evaluation function compared with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm is generated. It is well known that unknown inputs, control inputs, and model uncertainties are coupled in many industrial systems, which are the sources of false alarms and can corrupt the performance of the FDI system. This means that FDI systems have to be sensitive to faults and simultaneously robust to the unknown inputs and the model uncertainties. Therefore it is of great significance to design a robust FDI system that provides both sensitivity to faults and robustness to the unknown inputs and the model uncertainties, that is, the robust FDI issue, see for example, [39,40]. Recently, an H_∞-filtering formulation has been presented to solve the robust FDI and robust fault detection filter (RFDF) design problems for switched systems (see, e.g., [41–47] and the references therein). However, to the best of our knowledge, the RFDF problem of SPLSs has not been fully investigated. Moreover, the method given in [41–47] cannot be applied to SPLSs, and this constitutes the main motivation of the present study.

In this paper, we are interested in dealing with the problem of RFDF by constructing an appropriate multiple co-positive type Lyapunov–Krasovskii functional as well as applying the average dwell time approach for SPLSs with time-varying delays. The main contributions of this paper can be summarized as follows: (i) the residual generator is constructed based on the filter, and the design of RFDF is formulated as positive L_1 filtering problem.

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The objective is to make the error between the fault and the residual as small as possible, and increase robustness of the residual to the unknown inputs and the model uncertainties; (ii) by using the average dwell time approach, sufficient conditions for the existence of such filter are established in terms of linear matrix inequalities (LMIs). The parameterized matrices of this filter are constructed by solving the corresponding LMIs; and (iii) two simulation examples are presented to demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section 2, system formulation and some necessary lemmas are given. In Section 3, a sufficient condition for the existence of L_1-gain performance for SPLS with time-varying delay is established. Then based on the above result, the RFDF design problem is solved. Two numerical examples are provided to illustrate the design results in Section 4. Concluding remarks are given in Section 5.

1.1. Notations

In this paper, $A \succeq 0 (\preceq 0)$ means that all entries of matrix A are non-negative (non-positive); $A \succ 0 (\prec 0)$ means that all entries of A are positive (negative); $A \succ B (A \succeq B)$ means that $A - B \succ 0 (A - B \succeq 0)$. A^T means the transpose of the matrix A; $R(R_+)$ is the set of all real (positive real) numbers; $R^n(R^n_+)$ is the n-dimensional real (positive real) vector space; $R^{n\times k}$is the set of all real matrices of $(n\times k)$-dimension. $\|x\| = \sum_{k=1}^n |x_k|$, where x_k is the k-th element of $x \in R^n$. $L_1[t_0,\infty)$ is the space of absolute integrable vector-valued functions on $[t_0,\infty)$, i.e., we say $z:[t_0,\infty)\to R^k$ is in $L_1[t_0,\infty)$ if $\int_{t_0}^{\infty}\|z(t)\|\,dt < \infty$.

2. Problem statements and preliminaries

Consider the following switched linear systems with time-varying delays:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t) + E_{\sigma(t)}w(t) + G_{\sigma(t)}f(t), \\ y(t) = C_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-d(t)) + D_{\sigma(t)}u(t) + F_{\sigma(t)}w(t) + H_{\sigma(t)}f(t), \\ x(t_0+\theta) = \varphi(\theta),\ \theta \in [-h_2, 0], \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state, $y(t)\in R^q$ is the measured output; and $u(t)\in R^m$, $w(t)\in R^p$, $f(t)\in R^{\bar{r}}$ are the control input, disturbance input and the fault input, respectively, which belong to $L_1\ [0,\infty)$; $\sigma(t)\ :\ [0,\infty)\to \underline{N} = \{1,2,\cdots,N\}$ is the switching signal with N being the number of subsystems; A_i, A_{di}, B_i, E_i, G_i, C_i, C_{di}, D_i, F_i and H_i, $i\in\underline{N}$, are constant matrices with appropriate dimensions; $\varphi(\theta)$ is a vector-valued initial function defined on interval $[-h_2, 0]$, $h_2>0$; $t_0=0$ is the initial time, and t_κ denotes the κ-th switching instant; $d(t)$ is assumed to be the interval time-varying delay satisfying either of the following two cases:

$$0 \le h_1 \le d(t) \le h_2;\ \dot{d}(t) \le \tau < 1, \tag{C1}$$

$$0 \le h_1 \le d(t) \le h_2 \tag{C2}$$

where h_1 and h_2 are positive scalars representing the upper and the lower bounds of the time delay, respectively.

Remark 1. It is clear that (C2) contains (C1). The time-varying delay $d(t)$ is differentiable and bounded with a constant delay-derivative bound in (C1), whereas it is continuous and bounded in (C2). Then, in the sense, the criteria obtained under (C1) are less conservative than those under (C2). However, if the information of the derivation of time delay is unknown, only (C2) can be used to deal with the situation.

Definition 1. System (1) is said to be positive if, for any initial conditions $\varphi(\theta)\succeq 0$, $\theta\in[-h_2,0]$, $u(t)\succeq 0$, $w(t)\succeq 0$, $f(t)\succeq 0$ and any switching signals $\sigma(t)$, the corresponding state trajectory $x(t)\succeq 0$ and output $y(t)\succeq 0$ hold for all $t\ge t_0$.

Definition 2. [48] A is called a Metzler matrix, if the off-diagonal entries of matrix A are non-negative.

The following lemma can be obtained from Lemma 3 in [26] and Proposition 1 in [27].

Lemma 1. System (1) is positive if and only if A_i, $i\in\underline{N}$, are Metzler matrices, $A_{di}\succeq 0$, $B_i\succeq 0$, $E_i\succeq 0$, $G_i\succeq 0$, $C_i\succeq 0$, $C_{di}\succeq 0$, $D_i\succeq 0$, $F_i\succeq 0$, and $H_i\succeq 0$, $i\in\underline{N}$.

An FDI system consists of a residual generator and a residual evaluation stage including an evaluation function and a threshold. For the purpose of residual generation, the following fault detection filter is constructed as a residual generator

$$\begin{cases} \dot{\hat{x}}(t) = A_{f\sigma(t)}\hat{x}(t) + B_{f\sigma(t)}y(t), \\ r(t) = C_{f\sigma(t)}\hat{x}(t) + D_{f\sigma(t)}y(t), \end{cases} \tag{2}$$

where $\hat{x}(t)\in R^n$ and $r(t)\in R^l$ are the state and the residual, respectively. A_{fi}, B_{fi}, C_{fi} and D_{fi}, $i\in\underline{N}$, are the parameterized filter matrices to be determined.

Remark 2. The residual generator is a positive switched system, and the parameters of filter (2) depend on system modes. In our paper, the filter (2) is assumed to be switched synchronously with the switching signal in system (1). This means that the switching signal in filter (2) is the same as that in system (1).

For the purpose of fault detection, it is not necessary to estimate the fault $f(t)$. Similar to [40,41], a suitable weighting matrix $Q_f(s)$ was introduced to limit the frequency interval, in which the fault should be identified, and the system performance could be improved. Considering that the FDI problem is a special case of RFDF with $Q_f(s)=I$, our attentions will be only focused on the RFDF problem.

One minimal realization of $\hat{f}(s) = Q_f(s)f(s)$ is supposed to be

$$\begin{cases} \dot{\bar{x}}(t) = A_Q\bar{x}(t) + B_Qf(t), \\ \hat{f}(t) = C_Q\bar{x}(t) + D_Qf(t), \end{cases} \tag{3}$$

where $\bar{x}(t)\in R^{n_f}$ is the state of the weighted fault, $f(t)$ is the original fault and $\hat{f}(t)\in R^{\bar{r}}$ is the weighted fault. A_Q, B_Q, C_Q, D_Q are assumed to be known real constant matrices with appropriate dimensions.

Denoting $e(t) = r(t) - \hat{f}(t)$, and augmenting the model of system (1) to include the states of (2) and (3), we can obtain the augmented system as follows

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_{\sigma(t)}\tilde{x}(t) + \tilde{A}_{d\sigma(t)}\tilde{x}(t-d(t)) + \tilde{E}_{\sigma(t)}\tilde{w}(t), \\ e(t) = \tilde{C}_{\sigma(t)}\tilde{x}(t) + \tilde{C}_{d\sigma(t)}\tilde{x}(t-d(t)) + \tilde{F}_{\sigma(t)}\tilde{w}(t), \end{cases} \tag{4}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \\ \bar{x}(t) \end{bmatrix},\ \tilde{w}(t) = \begin{bmatrix} u(t) \\ w(t) \\ f(t) \end{bmatrix},\ \tilde{A}_i = \begin{bmatrix} A_i & 0 & 0 \\ B_{fi}C_i & A_{fi} & 0 \\ 0 & 0 & A_Q \end{bmatrix},\ \tilde{A}_{di} = \begin{bmatrix} A_{di} & 0 & 0 \\ B_{fi}C_{di} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{E}_i = \begin{bmatrix} B_i & E_i & G_i \\ B_{fi}D_i & B_{fi}F_i & B_{fi}H_i \\ 0 & 0 & B_Q \end{bmatrix},\ \tilde{C}_i = \begin{bmatrix} D_{fi}C_i & C_{fi} & -C_Q \end{bmatrix},\ \tilde{C}_{di} = \begin{bmatrix} D_{fi}C_{di} & 0 & 0 \end{bmatrix}$$

$$\tilde{F}_i = \begin{bmatrix} D_{fi}D_i & D_{fi}F_i & D_{fi}H_i - D_Q \end{bmatrix},\ i\in\underline{N}.$$

Definition 3. [28] System (4) is said to be exponentially stable under switching signal $\sigma(t)$, if there exist constants $\alpha\ge 1$ and $\beta>0$, such that the solution $\tilde{x}(t)$ of system (4) satisfies $\|\tilde{x}(t)\| \le \alpha\|\tilde{x}(t_0)\|_C e^{-\beta(t-t_0)}$, $\forall t\ge t_0$, where $\|\tilde{x}(t_0)\|_C = \sup_{t_0-h_2\le\delta\le t_0}\{\|\tilde{x}(\delta)\|\}$.