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Research Article

Improved delay-dependent stability of neutral type neural networks with distributed delays

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A R T I C L E I N F O

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A B S T R A C T

This paper deals with the problem of improved delay-dependent stability analysis of neutral type neural networks with distributed delays. These conditions are in terms of linear matrix inequality (LMI), easily checked by recently developed algorithms in solving linear matrix inequalities (LMIs). Finally, numerical examples demonstrate effectiveness of the proposed method.

1. Introduction

Time delay often occurs in many practical systems, leading to instability, and poor performance of systems. Problems of time delay systems have thus undergone much stability analysis in recent decades [1–5,7,9,11,12,14,15,17–23,25–29], especially the stability problem of neutral delay-differential systems [7,18–20,28]. Over recent years, dynamical neural networks have attracted considerable attention because of potential applications in various signal processing problems: e.g. optimization, image processing, associative memory design. Some applications require uniqueness and asymptotic stability of the equilibrium point of a designed neural network. Stability analysis of neural networks has been probed and various types of stability conditions proposed in literature [6,8,10,12,13,15,16,21], and/or references therein. On the other hand, time delays are encountered in implementation of artificial neural networks [10,21], often causing oscillation and instability of a neural network. This motivates the study of stability of delayed neural networks. Problems of stability of delayed neural networks have been extensively investigated and stability results from various kinds of delayed neural networks presented in [1,4,5,11,12,15,23,24,26].

Due to complicated dynamic properties of neural cells in the real world, existing neural network models in many cases fail to characterize properties of neural reaction process precisely. These systems should contain some information on derivatives of the past state to model dynamics for such complex neural reactions. Stability analysis of neural networks of the neutral type has thus received considerable attention in recent years [2,7,17–20,25,28,29]. Recently, delay-dependent stability criteria for a class of neutral equations describing neural networks are investigated in [25,29]. Such criteria are less conservative than delay-independent counterparts for delay sufficiently small. However, the sufficient criterion in [25,29] is only suitable for application to the particular class of neutral-type cellular neural networks (CNNs) with constant amplification functions. Although the results in Liu [15] are less conservative than some existing ones, they can be improved by employing the integral inequality approach to get a larger hound for discrete delays. Furthermore, neutral type and distributed delay are not considered in Liu [15].

On the other hand, note that while signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that distributed delays are incorporated into the model. Many scientific and technical workers have joined the study field with great interest, with various interesting results on asymptotic stability of neural networks with constant delay or without delay reported [20,27]. As is well known, constant fixed delay in models of delayed feedback provides a good approximation in simple circuits consisting of a small number of cells. Still, neural networks usually have a spatial extent due to the presence of multiple parallel pathways with variant axon sizes and lengths. There will be

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distribution of conduction velocities along these pathways and distribution of propagation delays. In these circumstances, signal propagation is not instantaneous and cannot be modeled with discrete delays; a more appropriate way is to incorporate continuously distributed delays. It should be mentioned that most recently, the global asymptotic stability analysis problem has been investigated in [7,12,20,27] for a general class of neural networks with both discrete and distributed delays, where a linear matrix inequality (LMI) approach has been developed to establish the sufficient stability conditions. For neural networks in neutral type with unbounded distributed delays, some stability conditions were given in [24]. It is noted that stability results in [20,27] are delay-independent. Usually, delay-dependent stability results are less conservative than delay-independent ones, especially when the delay size is small [9,14]. Therefore, stability results in [20,27] are conservative, which can be improved by developing delay-dependent stability results. So far there are only a few papers that have taken neutral-type phenomenon into account in delayed neural networks [7,12,17–19,28]. Practically, such phenomenon always appears in study of automatic control, population dynamics and vibrating masses attached to an elastic bar, etc.

Motivated by the statement above, this paper proposes improved delay-dependent stability of the neural type network with distributed delay studied. By employing Lyapunov–Krasovskii functional form based on the Leibniz–Newton formula, new delay-dependent stability conditions arise in terms of integral inequality approach (IIA). Maximum allowable delay bound (MADB) $\overline{h}$ can be computed by solving a quasi-convex optimization problem. Compared with existing stability results, derived condition is easier to check by resorting to recently developed standard algorithms in solving LMIs such as interior-point methods, with no tuning of parameters involved [3]. Four examples illustrate effectiveness and applicability of the proposed method.

2. Problem formulation

Consider a class of neutral type delayed neural networks with both discrete and distributed delays described by

$$\dot{u}_i(t) = -c_i u_i(t) + \sum_{i=1}^{n} a_{0ij} f_j(u_j(t)) + \sum_{i=1}^{n} a_{1ij} f_j(u_j(t-h)) + \sum_{i=1}^{n} b_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(u_j(s)) ds + \sum_{i=1}^{n} d_{ij} \dot{u}_j(t-h) + J_i \quad (1a)$$

$$u_i(t) = \phi_i(t), \quad -\infty \le t \le 0 \quad (1b)$$

for $i = 1, 2, 3, ..., n$, integer $n \ge 1$ denotes number of units in the neural network; $u_i(t)$ is the state of ith unit at time t; c_i denotes the passive decay rate; a_{0ij}, a_{1ij}, b_{ij}, and d_{ij} are interconnection matrices representing weight coefficients of neurons; h as a constant scalar represents delay of the neural network; $\phi_i(t)$, is initial condition of the neural network. J_i , is external constant input; $f_j(\cdot)$, is activation function; delay kernel k_i is a real valued continuous nonnegative function defined on $[0, +\infty]$, assumed to satisfy $\int_0^{\infty} k_i(s) ds = 1$, for $i = 1, 2, 3, ..., n$.

Throughout the paper, we make the following assumption on the activation function in the delayed neural network (1a) and (1b).

Assumption 1. [23]. It is assumed that each activation function of $f_j (j = 1, 2, ..., n)$ possesses the following condition:

$$0 \le \frac{f_i(\varsigma_1) - f_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \le \gamma_i, \quad \varsigma_1 \ne \varsigma_2 \in R, \quad i = 1, 2, ..., n, \quad (2)$$

where $\gamma_i (i = 1, 2..., n)$ are known constant scalars.

Next, equilibrium point $u^* = [u_1^*, \cdots, u_n^*]^T$ of system (1a) and (1b) is shifted to the origin via transformation $x(t) = u(t) - u^*$, then system (1a) and (1b) is equivalently written as system

$$\dot{x}(t) - D\dot{x}(t-h) = -Cx(t) + A_0 g(x(t)) + A_1 g(x(t-h)) + B \int_{-\infty}^{t} K(t-s) g(x(s)) ds, \quad (3)$$

where $C = diag(c_1, ..., c_n)$, $A_0 = [a_{0ij}]_{n\times n}$, $A_1 = [a_{1ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$, $D = [b_{ij}]_{n\times n}$, $K(s) = diag(k_1(s), ..., k_n(s))$, $x(\cdot) = [x_1(\cdot), \cdots, x_n(\cdot)]^T$, $g(x(\cdot)) = [g_1(x_1(\cdot)), \ldots, g_n(x_n(\cdot))]^T$, $g_i(x_i(\cdot)) = f_i(x_i(\cdot) + u_i^*) - f_i(u_i^*)$, $i = 1, 2, ..., n$. Obviously, function $g_j(\cdot) (j = 1, 2..., n)$ satisfies the following condition:

$$0 \le \frac{g_i(x_i)}{x_i} \le \gamma_i, \quad g_i(0) = 0, \quad \forall x_i \ne 0, \quad i = 1, 2, ..., n, \quad (4)$$

equivalent to

$$g_i(x_i)(g_i(x_i) - \gamma_i x_i) \le 0, \quad g_i(0) = 0, \quad \forall x_i \ne 0, \quad i = 1, 2, ..., n. \quad (5)$$

First, we introduce the following lemma, used in the proof of our main results.

Lemma 1. [9]. For any constant matrix $E \in R^{m\times m}$, with $E > 0$, scalars $b > a$, vector function $\omega : [a, b] \to R^m$ such that integrations in the following are well-defined, then

$$(b-a) \int_a^b \omega^T(\theta) E \omega(\theta) d\theta \ge \left[\int_a^b \omega(\theta) d\theta \right]^T \left[\int_a^b \omega(\theta) d\theta \right] \quad (6)$$

Lemma 2. [15]. For any positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \ge 0, \quad (7a)$$

the following integral inequality holds:

$$-\int_{t-h}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) ds \le \int_{t-h}^{t} [x^T(t) x^T(t-h) \dot{x}^T(s)] \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds. \quad (7b)$$

Lemma 3. [3]. The following matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0, \quad (8a)$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depend affine on x, is equivalent to

$$R(x) < 0, \quad (8b)$$

$$Q(x) < 0, \quad (8c)$$

and

$$Q(x) - S(x) R^{-1}(x) S^T(x) < 0. \quad (8d)$$

Based on the Lyapunov–Krasovskii stability theorem and integral inequality approach (IIA), the following result is obtained.

Theorem 1. Let $h > 0$ be a given scalar. Then, under Assumption 1, the origin is the unique equilibrium point of the delayed neural network in (3), which is globally asymptotically stable for any h satisfying $0 < h \le \overline{h}$ if there exist symmetry positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $W = W^T > 0$, $U = U^T > 0$,