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Research Article

Further improvement on delay-range-dependent stability results for linear systems with interval time-varying delays

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ABSTRACT

This paper provides an improved delay-range-dependent stability criterion for linear systems with interval time-varying delays. No model transformation and no slack matrix variable are introduced. Furthermore, overly bounding for some cross term is avoided. The resulting criterion has advantages over some previous ones in that it involves fewer matrix variables but has less conservatism, which is established theoretically. Finally, two numerical examples are given to show the effectiveness of the proposed results.

1. Introduction

Time delay is encountered in many dynamic systems such as chemical or process control systems and networked control systems and often results in poor performance and can lead to instability [1]. During the last decade, increasing research interest has been paid to the stability analysis and control design of the time delay systems [2–20]. Recently, to reduce the conservatism, many methods were adopted: descriptor model transform [2], inequality bounding technique method [13,14], integral inequality method [12,17], free weighting matrices [5], Jensen's inequality [15,16,18], and delay decomposition [12]. From this it is seen that the integral inequality method may have some potential in the study of delay-dependent stability.

Recently, a special type of time delay in practical engineering systems, that is interval time-varying delay, is investigated [5–7, 9–10,15–19]. The characteristic of interval time-varying delay is that time delay can vary in an interval in which the lower bound is not restricted to be 0. The typical examples of systems with interval time-varying delay are networked control systems, chemical process and flight systems. A latest stability condition for the interval time delay case is provided in [5–7,9–10,15–19], but there is room for further investigation. As regards the stability result in Jiang et al. [7], it is only useful for fast time-varying delay. By contrast, with the free weighting matrix method He et al. [5] presented stability criteria as diverse as those for slow or fast delay. Nevertheless the criteria still leave some room for improvement in accuracy as well as complexity due to the method used. Very recently the results in He et al. [5] have been further improved in Shao [17] where a new Lyapunov functional is constructed and fewer matrix variables are involved. Nevertheless the criteria still leave some room for improvement in accuracy as well as complexity due to the method used.

In this paper, we study the stability problem for systems with time-varying delay in a range by choosing an appropriate Lyapunov functional. Based on such stability conditions are derived via the Lyapunov–Krasovskii functional combining with LMI techniques and integral inequality approach to express the relationships between the terms of the Leibniz–Newton formula, and simple and delay-range-dependent stability criteria are also derived. The advantage of the criterion lies in its simplicity and less conservatism. This is established theoretically. Examples are also given to further illustrate the reduced conservatism of the stability result.

2. Stability analysis

Consider the following linear systems with an interval time-varying delay:

$$\dot{x}(t) = Ax(t) + Bx(t-h(t)) \quad t > 0 \tag{1a}$$

$$x(t) = \phi(t), t \in [-h_2, 0] \tag{1b}$$

where $x(t) \in R^n$ is the state vector of the system; $A, B \in R^{n \times n}$ are constant matrices; and $\varphi(t)$is a continuously real-valued initial function. $h(t)$is a time-varying continuous function. Throughout this paper, we will analyze the following two scenarios of the time-varying delay $h(t)$.

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Case 1. $h(t)$ is a differentiable function satisfying

$$0 \le h_1 \le h(t) \le h_2, \quad |\dot{h}(t)| \le h_d, \quad \forall t \ge 0. \tag{2}$$

Case 2. $h(t)$ is a differentiable function satisfying

$$0 \le h_1 \le h(t) \le h_2, \tag{3}$$

where h_1 and h_2 are the lower and upper delay bounds, respectively, and h_1, h_2 and h_d are constants.

In the following, we will develop some practically computable stability criteria for the system described (1a and b).The following lemmas are useful in deriving the criteria. First, we introduce the following technical Lemma 1 of integral inequality approach (IIA).

Lemma 1 [12]. *For any positive semi-definite matrices*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \ge 0. \tag{4}$$

Then, we obtain

$$-\int_{t-h}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) ds \le \int_{t-h}^{t} \begin{bmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds \tag{5}$$

Lemma 2 [1]. *The following matrix inequality,*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0, \tag{6a}$$

where $Q(x) = Q^{-T}(x)$, $R(x) = R^T(x)$ and $S(x)$ depend on affine on x, is equivalent to

$$R(x) < 0, \tag{6b}$$

$$Q(x) < 0, \tag{6c}$$

and

$$Q(x) - S(x) R^{-1}(x) S^T(x) < 0. \tag{6d}$$

In this paper, a new Lyapunov functional is constructed, which contains the information of the lower bound of delay h_1 and upper bound h_2. The following theorem presents a delay-range-dependent result in terms of LMIs and expresses the relationships between the terms of the Leibniz–Newton formula.

Theorem 1. *For given scalars h_1, h_2, and h_d, the system (1a and b) subject to (2) is asymptotically stable if there exist $P = P^T > 0$, $Q_i = Q_i^T > 0$, $R_j = R_j^T > 0$, $(i = 1,2,3; j = 1,2)$*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \ge 0, \; Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} \ge 0 \text{ and}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12}^T & Z_{22} & Z_{23} \\ Z_{13}^T & Z_{23}^T & Z_{33} \end{bmatrix} \ge 0,$$

such that these LMIs hold:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ 0 & \Xi_{23}^T & \Xi_{33} & 0 & 0 & 0 \\ \Xi_{14}^T & \Xi_{24}^T & 0 & \Xi_{44} & 0 & 0 \\ \Xi_{15}^T & \Xi_{25}^T & 0 & 0 & \Xi_{55} & 0 \\ \Xi_{16}^T & \Xi_{26}^T & 0 & 0 & 0 & \Xi_{66} \end{bmatrix} < 0, \tag{7a}$$

$$R_1 - X_{33} \ge 0, \tag{7b}$$

$$R_2 - Y_{33} \ge 0, \tag{7c}$$

$$R_2 - Z_{33} \ge 0, \tag{7d}$$

where

$$\begin{aligned}
&\Xi_{11} = A^T P + PA + Q_1 + Q_2 + Q_3 + \delta X_{11} + X_{13} \\
&\quad + X_{13}^T, \Xi_{12} = PB, \Xi_{14} = \delta X_{12} - X_{13} + X_{23}^T, \\
&\Xi_{15} = \delta A^T R_1, \Xi_{16} = (h_2 - \delta) A^T R_2, \\
&\Xi_{22} = -(1-h_d) Q_3 + (h_2 - \delta) Y_{22} - Y_{23} - Y_{23}^T \\
&\quad + (h_2 - \delta) Z_{11} + Z_{13} + Z_{13}^T, \\
&\Xi_{23} = (h_2 - \delta) Z_{12} - Z_{13} + Z_{23}^T, \\
&\quad \Xi_{24} = (h_2 - \delta) Y_{12}^T - Y_{13}^T + Y_{23}, \Xi_{25} = \delta B^T R_1, \\
&\Xi_{26} = (h_2 - \delta) B^T R_2, \Xi_{33} = -Q_2 + (h_2 - \delta) Z_{22} - Z_{23} - Z_{23}^T, \\
&\Xi_{44} = -Q_1 + (h_2 - \delta) Y_{11} + Y_{13} + Y_{13}^T + \delta X_{22} - X_{23} - X_{23}^T, \\
&\Xi_{55} = -\delta R_1, \Xi_{66} = -(h_2 - \delta) R_2.
\end{aligned}$$

Proof. If we can prove that Theorem 1 holds for two cases, $\delta \le h(t) \le h_2$ and $h_1 \le h(t) \le \delta$, where $\delta = (h_2 + h_1)/2$, then Theorem 1 is true.

Case 1. When $\alpha\delta \le h(t) \le h_2$

Construct a Lyapunov–Krasovskii functional candidate as

$$\begin{aligned}
V(x_t) &= x^T(t) P x(t) + \int_{t-\delta}^{t} x^T(s) Q_1 x(s) ds \\
&\quad + \int_{t-h_2}^{t} x^T(s) Q_2 x(s) ds + \int_{t-h(t)}^{t} x^T(s) Q_3 x(s) ds \\
&\quad + \int_{-\delta}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta + \int_{-h_2}^{-\delta} \int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta.
\end{aligned} \tag{8}$$

Taking time derivative $V(x_t)$ for $t \in [0, \infty)$ along trajectory (1a and b) yields

$$\begin{aligned}
\dot{V}(x_t) &= x^T(t)\left(PA + A^T P\right) x(t) + x^T(t) PBx(t-h(t)) + x^T(t-h(t)) B^T P x(t) \\
&\quad + x^T(t)(Q_1 + Q_2 + Q_3) x(t) - x^T(t-\delta) Q_1 x(t-\delta) \\
&\quad - x^T(t-h_2) Q_2 x(t-h_2) - x^T(t-h(t))(1-\dot{h}(t)) Q_3 x(t-h(t)) \\
&\quad + \dot{x}^T(t) \delta R_1 \dot{x}(t) + \dot{x}^T(t)(h_2-\delta) R_2 \dot{x}(t) \\
&\quad - \int_{t-\delta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds - \int_{t-h_2}^{t-\delta} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
&\le x^T(t)(PA + A^T P) x(t) + x^T(t) PBx(t-h(t)) + x^T(t-h(t)) B^T P x(t) \\
&\quad + x^T(t)(Q_1 + Q_2 + Q_3) x(t) - x^T(t-\delta) Q_1 x(t-\delta) \\
&\quad - x^T(t-h_2) Q_2 x(t-h_2) \\
&\quad - x^T(t-h(t))(1-h_d) Q_3 x(t-h(t)) + \dot{x}^T(t)\left[\delta R_1 + (h_2-\delta) R_2\right] \dot{x}(t) \\
&\quad - \int_{t-\delta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds - \int_{t-h_2}^{t-\delta} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
&= x^T(t)(PA + A^T P) x(t) + x^T(t) PBx(t-h(t)) + x^T(t-h(t)) B^T P x(t) \\
&\quad + x^T(t)(Q_1 + Q_2 + Q_3) x(t) - x^T(t-\delta) Q_1 x(t-\delta) \\
&\quad - x^T(t-h_2) Q_2 x(t-h_2) - x^T(t-h(t))(1-h_d) Q_3 x(t-h(t)) \\
&\quad + \dot{x}^T(t)\left[\delta R_1 + (h_2-\delta) R_2\right] \dot{x}(t) \\
&\quad - \int_{t-\delta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds - \int_{t-h(t)}^{t-\delta} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
&\quad - \int_{t-h_2}^{t-h(t)} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
&= x^T(t)(PA + A^T P) x(t) + x^T(t) PBx(t-h(t)) + x^T(t-h(t)) B^T P x(t) \\
&\quad + x^T(t)(Q_1 + Q_2 + Q_3) x(t) - x^T(t-\delta) Q_1 x(t-\delta) \\
&\quad - x^T(t-h_2) Q_2 x(t-h_2) - x^T(t-h(t))(1-h_d) Q_3 x(t-h(t)) \\
&\quad + \dot{x}^T(t)[\delta R_1 + (h_2-\delta) R_2] \dot{x}(t) \\
&\quad - \int_{t-\delta}^{t} \dot{x}^T(s)(R_1 - X_{33}) \dot{x}(s) ds - \int_{t-h(t)}^{t-\delta} \dot{x}^T(s)(R_2 - Y_{33}) \dot{x}(s) ds
\end{aligned}$$