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Research Article

Adaptive synchronization and parameter identification of chaotic system with unknown parameters and mixed delays based on a special matrix structure

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ABSTRACT

In this paper, we investigate the synchronization and parameter identification of chaotic system with unknown parameters and mixed delays. A new approach is proposed for designing a controller and a update rule of unknown parameters based on a special matrix structure, and the synchronization and the parameter identification are realized under the controller and the update rule. Numerical simulations are carried out to confirm the effectiveness of the approach. A significant advantage is that the process of designing a controller and a update rule become very clear and easy by the proposed approach.

1. Introduction

Chaotic systems are very complex dynamical systems that possess some special attributes such as extremely sensitivity to initial conditions, broad fourier transform spectra, fractal properties of the motion in phase space and strange attractors. Due to its potential applications such as secure communication, biological systems, information science, etc., since Pecora and Carroll [1] introduced a method to synchronize two identical chaotic systems with different initial conditions, much attention has been payed to control and chaos synchronization. In the past two decades, a variety of approaches have been proposed for the synchronization of chaotic systems such as lag-synchronization [2], adaptive synchronization [3], the output feedback H_∞ synchronization [4,5], the L_2-L_∞ synchronization [6], projective synchronization [7,8], phase synchronization [9], generalized synchronization (GS) [10], etc.

Most of the theoretical results concerning synchronization of chaotic systems mainly focus on the systems whose models are identical or similar, and parameters are exactly known in advance. But in many practical situations, the parameters of many systems cannot be known entirely. How to effectively synchronize two chaotic systems with unknown parameters is an important problem for the theoretical and practical applications.

From the viewpoint of engineering applications and characteristics of channel, a time delay always exists. The systems with time delay are difficult to achieve satisfactory performance. So, the stability issue of time-delay systems is of practical importance. Since Peng first found chaos in time-delayed systems [11], the time-delay chaotic systems have received more attention [12–17]. However, effected by multiple physical process, some systems may be mixed delay nonlinear systems with multiple time delays. And the stability and control theorem of thus system becomes more complicated and difficult.

In fact, the parameters of some nonlinear system with mixed delay may also be unknown. How to synchronize nonlinear system with mixed delay and unknown parameter is a more complex task. Although some progresses have been made in this area [18–23], the authors generally give a controller and a update rule for a certain system and do not introduce the designing process of the controller and the update rule. For most readers, in order to solve other similarly problem, they pay more attention to studying and mastering the designing approach of the controller and the update rule than the results in a research paper.

For this purpose, we propose a simple approach of designing a controller and a update rule to synchronize chaotic system with

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mixed delay and unknown parameters. We can not only realize synchronization but also recognize unknown parameters, and the processing of designing the controller and the update rule of unknown parameters is also very easy to be understood and mastered.

This paper is organized as follows: in Sections 2, we introduce the new approach and prove it; in Section 3, we take synchronizing hyperchaotic Lü system with mixed time delay and unknown parameters as an example to explain how to use the proposed approach; Finally, a conclusion is made in Section 4.

2. The main result

A general nonlinear system with mixed time delays consisting of unknown parameters, which is described as follows:

$$\dot{x}(t)=f(x(t))+\varphi(x(t))+g_1(x(t-\tau_1))+g_2(x(t-\tau_2))+\cdots +g_k(x(t-\tau_k)),\quad k\in N^+ \tag{1}$$

where $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T\in R^n$ are the state variables of the nonlinear system at time t, the positive constants τ_1, τ_2, …, $\tau_k(\tau_i>0, i=1,2,\dots,k)$ are the time delays; and function $f(x(t))$, $\varphi(x(t))$, $g_1(x(t-\tau_1))$, $g_2(x(t-\tau_2)),\dots,g_k(x(t-\tau_k))$ are real-valued continuous functions satisfying the Lipschitz condition and the function $\varphi(x(t))$ including unknown parameters.

We can transform system (1) as

$$\dot{x}(t)=A(x(t))x(t)+B\eta(x(t))+G_1x(t-\tau_1)+G_2x(t-\tau_2)+\cdots +G_kx(t-\tau_k),\quad k\in N^+ \tag{2}$$

where $A(x(t))$ is $R^{n\times n}$ parameter matrix including state variables, G_1, G_2, …, G_k are $R^{n\times n}$ matrixes and $f(x(t))=A(x(t))x(t)$, $\varphi(x(t))=B\eta(x(t))$, $g_1(x(t-\tau_1))=G_1x(t-\tau_1)$, $g_2(x(t-\tau_2))=G_2x(t-\tau_2)$, …, $g_n(x(t-\tau_n))=G_nx(t-\tau_n)$ and matrix B includes unknown parameters. The key problem is how to design the controller $u(t)$ and the update rules of unknown parameters to make system (1) stable to zero.

Suppose matrix $\tilde{B}$ as the estimate of matrix B with unknown parameters and the unknown parameters respectively as $p_1(t)$, $p_2(t)$, …, $p_k(t)$, $k\in N^+$. Define the unknown parameters as parameter vector $p(t)=[p_1(t),p_2(t),\dots,p_k(t)]^T$, and the estimate value of unknown parameter vector as: $\tilde{p}(t)=[\tilde{p_1}(t),\tilde{p_2}(t),\dots,\tilde{p_k}(t)]^T$. Define $B_e=\tilde{B}-B$ and the parameter error as $e_p=\tilde{p}(t)-p(t)$. In order to control system (1), we design a controller $u(t)$ and get

$$\dot{x}(t)=A(x(t))x(t)+\tilde{B}\eta(x(t))+G_1x(t-\tau_1)+G_2x(t-\tau_2)+\cdots +G_kx(t-\tau_k)-B_e\eta(x(t))+u(t),\quad k\in N^+ \tag{3}$$

We can express $B_e\eta(x(t))$ by e_p

$$B_e\eta(x(t))=-\Psi e_p \tag{4}$$

where $\Psi\in R^{n\times k}$.

The key problem is how to design the controller $u(t)$ and the update rule of the unknown parameters. We suppose the controller as $u(t)=D(x(t))x(t)$ and the update rule as $\dot{e}_p=\Theta x(t)$, where $D(x(t))\in R^{n\times n}$ and $\Theta\in R^{k\times n}$. If we can design the matrix $D(x(t))$ and the matrix Θ, the controller $u(t)$ and the update rule $\dot{e}_p$ can also be realized.

Define $A(x(t))x(t)+\tilde{B}\eta(x(t))+D(x(t))x(t)=C(x(t))x(t)$ and we can get

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}_p\end{bmatrix}=\begin{bmatrix}C(x(t)) & \Psi\\ \Theta & 0\end{bmatrix}\begin{bmatrix}x(t)\\ e_p\end{bmatrix}+\begin{bmatrix}G_1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t-\tau_1)\\ 0\end{bmatrix} +\begin{bmatrix}G_2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t-\tau_2)\\ 0\end{bmatrix}+\cdots+\begin{bmatrix}G_k & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t-\tau_k)\\ 0\end{bmatrix} \tag{5}$$

Define

$$C(x(t))=\begin{bmatrix}c_{11} & c_{12} & \cdots & c_{1n}\\ c_{21} & c_{22} & \cdots & c_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ c_{n1} & c_{n2} & \cdots & c_{nn}\end{bmatrix} \tag{6}$$

and

$$G_i=\begin{bmatrix}g_{i,11} & g_{i,12} & \cdots & g_{i,1n}\\ g_{i,21} & g_{i,22} & \cdots & g_{i,2n}\\ \vdots & \vdots & \ddots & \vdots\\ g_{i,n1} & g_{i,n2} & \cdots & g_{i,nn}\end{bmatrix}\quad i=1,2,\dots,k \tag{7}$$

We study how to design the matrix $C(x(t))$ and the matrix Θ to solve the key problem of designing the controller $u(t)$ and the update rule $\dot{e}_p$. For this purpose, we propose an approach based on a special matrix as follows.

Theorem 1. *If the matrix $C(x(t)x$ and the matrix Θ in formula* (5) *satisfy*:

(1) $\Theta=-\Psi^T$.
(2) $c_{ij}=-c_{ji}\quad (i\neq j)$
(3) $c_{ii}+(\sum_{l=1}^{k}\sum_{j=1}^{n}(|g_{l,ij}|+|g_{l,ji}|))/2\le 0,\quad i=1,2,\dots,n$ *(not all* $c_{ii}+(\sum_{l=1}^{k}\sum_{j=1}^{n}(|g_{l,ij}|+|g_{l,ji}|))/2$ *equal to zero).*

The controlled system (3) *is stable to zero and the unknown parameter can be recognized.*

Proof. Construct a positive define Lyapunov function as follows:

$$V_1=x^T(t)x(t)+e_p^Te_p \tag{8}$$

Calculating the time derivative of the function (8) along the trajectory of system (5), we arrive at

$$\dot{V}_1=2x^T(t)C(x(t))x(t)+2\sum_{j=1}^{n}\sum_{i=1}^{n}g_{1,ij}(x_i(t)x_j(t-\tau_1)) +2\sum_{j=1}^{n}\sum_{i=1}^{n}g_{2,ij}(x_i(t)x_j(t-\tau_2))+\cdots +2\sum_{j=1}^{n}\sum_{i=1}^{n}g_{k,ij}(x_i(t)x_j(t-\tau_k)) \tag{9}$$

Because $2g_{l,ij}x_i(t)x_j(t-\tau_l)\le|g_{l,ij}|(x_i^2(t)+x_i^2(t-\tau_l))$, $l=1,2,\dots,k$, we can get

$$\dot{V}_1\le 2x^T(t)C(x(t))x(t)+\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{1,ij}|(x_i^2(t)+x_j^2(t-\tau_1)) +\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{2,ij}|(x_i^2(t)+x_j^2(t-\tau_2))+\cdots +\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{k,ij}|(x_i^2(t)+x_j^2(t-\tau_k)) \tag{10}$$

Hence, we construct a Lyapunov–Krasovskii function

$$V=x^T(t)x(t)+e_p^Te_p+\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{1,ij}|\int_{t-\tau_1}^{t}x_j^2(\varepsilon)\,d\varepsilon +\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{2,ij}|\int_{t-\tau_2}^{t}x_j^2(\varepsilon)\,d\varepsilon+\cdots+\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{k,ij}|\int_{t-\tau_k}^{t}x_j^2(\varepsilon)\,d\varepsilon \tag{11}$$

Calculating the time derivative of the Lyapunov–Krasovskii function (11) and we can get

$$\dot{V}_1\le 2x^T(t)C(x(t))x(t)+\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{1,ij}|(x_i^2(t)+x_j^2(t-\tau_1)) +\sum_{j=1}^{n}\sum_{i=1}^{n}|g_{1,ij}|(x_j^2(t)-x_j^2(t-\tau_1))$$